One Team Must Win, the Other Need Only Not Lose:

An Experimental Study of an Asymmetric Participation Game

Gary Bornstein, Tamar Kugler and Shmuel Zamir[1]

Center for the Study of Rationality and Interactive Decision Theory

The Hebrew University of Jerusalem

The research was supported by grants from the Israel Science Foundation (2001-2005) and the EU-TMR ENDEAR Research Network (FMRX-CT98-0238).

Please address correspondence to Gary Bornstein, Department of Psychology, the Hebrew University, Jerusalem 91905, Israel. E-Mail: MSGARY@mscc.huji.ac.il

[1] Also CNRS-EUREQua and CREST-LEI, FRANCE.

Abstract: Consider a parliamentary committee with an equal number of coalition and opposition members. The opposition needs a strict majority to pass a motion, whereas for the coalition a tie is sufficient to block the motion and maintain the status quo. Passing or blocking the motion is a public good shared equally by all members of the winning group, and voting is voluntary and costly. The members of which group are more likely to vote? To answer this question, we studied an asymmetric participation game where a tie favors one prespecified group over the other. The theoretical analysis of this game yielded two qualitatively different predictions, one in which members of the coalition are *slightly* more likely to participate than members of the opposition, and another in which members of the opposition are *much* more likely to participate than members of the coalition. The experimental results clearly support the first prediction.

Key words: Strategic Decision Making, Intergroup Competition, Voting Behavior, Participation Games.

1. Introduction

In this paper we study a type of competition between two groups in which the group whose members contribute more toward their collective group effort wins the competition and receives a reward. The reward is a public good which is non-excludable to individual members of the winning group, regardless of whether or not they contributed. Since individual contribution is assumed to be costly and voluntary, a problem of free-riding is created within each of the competing groups (Bornstein, 2003).

Past research on intergroup conflicts over public goods has focused mainly on the symmetric case where, if the competition is tied, the reward is divided equally between the two groups, or awarded to one of the groups at random (Palfrey & Rosenthal, 1983; Rapoport & Bornstein, 1987; Bornstein & Rapoport, 1988, Bornstein, 1992, Schram & Sonnemans, 1996ab). In the current investigation we consider an asymmetric competition where, is case of a tie, one pre-specified group receives the reward while the other receives nothing. In other words, to receive the public good one group must strictly win the competition while the other group need only not lose it.

To motivate this line of inquiry consider a parliamentary committee with an equal number of coalition and opposition members. The opposition intends to pass a motion to which the coalition objects. To pass the motion, the opposition needs a strict majority, whereas for the coalition a tie is sufficient to block the motion and maintain the status quo. The members of both groups have to decide whether to attend the meeting and vote according to their preferences. Attending the meeting is voluntary and costly (e.g., committee members have to forgo other, more personally beneficial, political activities). The public good – passing or blocking the motion -- is shared equally by all members of the winning group, whether or not they attended the meeting.

Such asymmetric competition raises an interesting question: The members of which group, the group that wins in case of a tie or the one that loses, are more likely to contribute toward their group's effort? To answer this question we conducted a theoretical and experimental study of an asymmetric participation game (Palfrey & Rosenthal, 1983). The game involves two competing groups with three members in each group. Each group member receives an initial endowment of e (e>0) and has to decide whether or not to contribute it. Players keep their endowment if they do not contribute, and contributions are not refunded regardless of the outcome of the game.[2] The decisions are made simultaneously, with no opportunity to communicate within or between the groups. The group with more contributions wins the competition and each of its members receives a reward of r (r>e). The reward is given to all the members of the winning group regardless of whether they contribute or not (hence the public good nature of the reward). The members of the losing group are paid nothing. The asymmetry is created by selecting one group beforehand by tossing a fair coin to be the winning group in case of a tie. The identity of this group is known to members of both groups.

We compared this asymmetric game to a symmetric one where players first make their decisions, and, *if* the game is tied, one group is selected by a toss of a fair coin to be the winner and each of its members receives a reward of r. The members of the other group receive nothing (Rapoport & Bornstein, 1987). Thus, in both the asymmetric and symmetric games, the winner in case of a tie is determined by a toss

[2] When the individual decision is binary the game can be conceptualized as a participation game where each player decides between participation (which is costly) and non-participation. We shall use the terms contribution and participation interchangeably.

of a coin. The difference between the two games is that in the asymmetric game the coin is tossed to select the winner *before* the decisions are made (and the outcome is common knowledge), whereas in the symmetric game the coin is tossed (if necessary) to select the winner *after* the decisions are made. We thus refer to these two treatments as the *before* and *after* treatments, respectively.

Section 2 below specifies the Nash equilibria of the symmetric and asymmetric participation games. The following sections describe an experiment which examined actual choice behavior in the two games.

1.1 Theoretical Analysis

The equilibria for the one-stage symmetric and asymmetric participation games are computed for the specific payoff parameters used in the experiment, $e= 0.25$, and $r= 1$.[3] The general case with any payoff parameters e and r is analyzed in Appendix A.

The *after* treatment (the symmetric game) has a unique pure-strategy equilibrium in which all six players contribute. In this equilibrium the game is tied and one randomly selected group wins the reward. As no player gets to keep his or her endowment, the expected payoff per player is $r/2= 0.5$. This equilibrium is clearly collectively (Pareto) deficient. Collective efficiency is achieved when all players withhold their contribution. This would again result in a tie, but since all the players keep their endowments, each receives an expected payoff of $r/2+e= 0.75$. The efficient outcome is unstable, however, as each player can benefit from unilaterally contributing his or her endowment and securing a payoff of r (a sure payoff of 1). For

[3] The payoffs are in New Israeli Shekels. 4 NIS equaled about 1US$ when the experiment took place.

the set of parameters chosen for the present study the symmetric game has no symmetric equilibria in mixed strategies (see appendix A).[4]

In analyzing the *before* treatment we consider only equilibria that are symmetric within each group, that is, we assume that all the members of the "coalition" (i.e., the group which wins in case of a tie) contribute with the same probability p, and all the members of the "opposition" (i.e., the group which loses in case of a tie) contribute with a different probability q.[5] For the set of parameters chosen, the asymmetric game has three such equilibria. In one equilibrium, members of the coalition are somewhat more likely to contribute than members of the opposition (p=0.579; q=0.421). In the other two equilibria the opposition members are much more likely to contribute than the coalition members. In one of these equilibria the coalition members contribute with p=0.237 and the opposition members with q=0.763, and in the other, members of the coalition contribute with a probability of p=0.5, while members of the opposition contribute with certainty (q=1).

The expected payoffs associated with each of the three solutions also differ substantially. In the first equilibrium (p=0.579; q=0.421) the expected payoff is NIS 0.90 for a coalition member and NIS 0.36 for an opposition member. In the second equilibrium (p=0.237 and q=0.763) the expected payoff is NIS 0.35 for a coalition member and NIS 0.91 for a opposition member. Note that in both solutions the sum of p and q is 1.[6] As a consequence the sum of payoffs under these two equilibria is also constant (NIS 1.26). Thus, even though the two equilibria are rather different, none is payoff-dominant over the other. The third equilibrium (p=.5 and q=1) pays

---

[4] The symmetric game has no mixed-strategy equilibrium in which **all** players in both groups contribute with the same probability p. However, it does have mixed-strategy equilibria in which members of one group contribute with probability p, while the members of the other group contribute with a different probability q. Specifically, there are two such equilibria, one in which p=0.662 and q=0.338, and another in which p=0.338 and q=0.662.

[5] This assumption reflects the fact that ex-ante players within each group are undistinguishable.

NIS 0.25 for a coalition member and NIS 0.875 for an opposition member and is thus Pareto-dominated by the second equilibrium and socially dominated by both (i.e. it is deficient when comparing total payoffs to all players).

This theoretical analysis provides a useful baseline for evaluating the experimental results. First, based on this analysis, we hypothesize that there will be more individual contribution or participation in the symmetric game (the *after* treatment) than in the asymmetric game (the *before* treatment). Second, the analysis indicates that the asymmetric game has three different equilibria; in the first equilibrium the members of the coalition are slightly more likely to contribute than members of the opposition, while in the second and third equilibria the members of the opposition are *much* more likely to contribute than members of the coalition. Game theory does predict which of the three equilibria, if any, will be observed. This remains an empirical issue which our experiment may help resolve.

There is one more point that needs to be discussed before describing the experiment. The game-theoretic analysis above applies to the one-stage game, while in our experiment the game was repeated 100 times. In order to avoid the theoretical complications of a repeated game, each experimental session included 12 participants (that is, two unrelated competitions were played on each round) and the participants were re-matched randomly at the beginning of each round. This random-matching protocol effectively prevents the players from employing repeated-game strategies of reciprocation, while providing them with ample opportunity to learn the structure of the one-stage game and adapt their behavior accordingly (e.g., Smith, 1984; Harley, 1981; Selten, 1991, Roth & Erev, 1995).

---

[6] This result holds for all cases where $0 < e/r < 216/625$; see Appendix A.

2. Method

Subjects and design: The participants were 192 undergraduate students at the Hebrew University of Jerusalem. They were recruited by campus advertisements promising monetary reward for participation in a decision-making experiment. The participants were scheduled in 16 cohorts of 12, and were paid contingent on their decisions and the decisions of their counterparts. Eight independent sessions were conducted in each of the two experimental treatments.

Procedure: The experiment was held in a computerized laboratory. Upon arrival each participant received NIS 10 for showing up and was seated in separate cubicle facing a personal computer. The participants were given written instructions concerning the rules and payoffs of the game (see Appendix B) and were asked to listen to these instructions while the experimenter read them aloud. Then the participants were given a quiz to test their understanding. Their answers were checked by the experimenters and, when necessary, explanations were repeated. The participants were also told that to ensure the confidentiality of their decisions they would receive their payment in sealed envelopes and leave the laboratory one at a time with no opportunity to meet the other participants.

Participants played 100 rounds of the same game. The number of rounds to be played was made known in advance. At the beginning of *each* round the 12 individuals were randomly divided into three-player groups and each group was paired randomly with another group. This random-matching protocol was carefully explained to the participants. In the *before* treatment, one group in each pair was randomly selected to be the winning group in case of a tie and the identity of this group was made known to members of both groups. Each individual in the *before* treatment played 50 rounds as a member of a "coalition" group and 50 rounds as a member of an "opposition" group. The order of these rounds was randomized. In both the *before* and *after* conditions, each player was given an initial endowment of NIS 0.25 at the beginning of each round and had to decide between contributing his or her endowment and keeping it. Following the completion of a round, each player received feedback concerning (a) the total number of contributors in his or her group; (b) the total number of contributors in the competing group; (c) his or her earnings in this round; and (d) his or her cumulative earnings. Following the last round, the participants were debriefed on the rationale and purpose of the study. They were then paid and dismissed individually without the opportunity to meet the other participants.

3. Results

3.1 Contribution rates: We hypothesized, based on the theoretical analysis, that overall contribution would be higher in the *after* treatment than in the *before* treatment. Consistent with this hypothesis, we found that the mean number of contributions per player (summed over the 100 rounds of play) in the *after* treatment was 0.60, as compared with a contribution rate of 0.545 in the *before* treatment. However, this difference in contribution rates is only marginally significant (t_{14}=1.456, $p<0.08$, one-tailed test). In the *before* treatment members of the coalition (group Co) contributed at a rate of .58 and members of the opposition (group Op) at a rate of .51. This difference in contribution rates is statistically significant (t_7=2.37, $p<0.05$).

Next we examine the distribution of outcomes across the 200 stage-games played in each treatment (two separate games per round X 100 rounds). In 42.4% of the games played in the *before* treatment group Co had more contributors than group

Op, in 27.5% of the games group Op had more contributors than group Co, and in the remaining 30.1% of the games there was an equal number of contributors in both groups (meaning, of course, that group Co received the reward). Of the 200 games played in the *after* treatment, 68% ended with one group winning the competition and 32% resulted in a tie.

3.2 Equilibrium Selection: In the *before* treatment we considered three Nash equilibria, all assuming that the members of the same group contribute with the same probability. These equilibria are qualitatively quite different, as one predicts that the members of group Co will be slightly more likely to contribute than members of group Op, while the other two predict that members of group Op will be much more likely to contribute than members of group Co. The observed behavior described above clearly corresponds to the first equilibrium.

Figure 1 plots the mean contribution rates in groups Co and Op in each (20-round) block. As can be see in the figure, in the first 20 rounds of play members of group Co contributed with a probability of 0.63, which is higher than the 0.579 probability prescribed by the equilibrium. However, as the game progressed, the contribution rates of group Co decreased, as players presumably learned that, given the actual contribution rates in group Op, they could withhold contribution and still win or tie the game and get the reward.

To test whether the decreasing trend in contribution rate is statistically significant, we fitted a regression line for each of the eight independent sessions in this treatment to predict the contribution rate from the block number, and extracted the unstandardized B coefficients. Except for one session, all the B coefficients were negative, indicating a decrease in contribution rates over time. The mean B was –0.01 (sd=0.008), which is significantly different from zero ($t_{(7)}=-3.363$, $p<0.012$).

Members of group Op started out with a contribution rate of 0.47 in the first 20-round block, which is slightly higher than the 0.421 contribution rate that the equilibrium predicts, and their contribution level did not change much during the course of the game. The trend in contribution rates in the Op treatment is not significantly different from 0 (mean B= 0.002, sd=0.0017; $p_{(7)}$=0.35, ns).

<Insert Figure 1 about here>

The contribution rates in the *after* condition are presented in Figure 1 (the contribution rates in this condition are superimposed on the results of the *before* condition). As can be seen in this figure, the players contributed at a rate of .60 in the first 20-round block and maintained this contribution rate throughout the game. To explain this result, we computed the expected payoff of player *i* in the symmetric game, given that all the other players contribute with a probability of 0.6 (the observed probability of contribution). The best response for player *i* is to contribute with probability 1. His or her worst response is to withhold contribution. However, the difference in payoffs between the best and worst responses is only NIS 0.05 -- a negligible amount. In other words, player *i* has little incentive to change his or her behavior toward the equilibrium.[7]

3.4 Payoffs: The mean payoff per player is 0.60 per round in the *before* treatment and 0.615 in the *after* treatment. The mean bpayoff in the *after* condition is higher than the equilibrium payoff (i.e., 0.5), reflecting the fact that contribution rates were lower than predicted by the equilibrium. In the *before* condition the mean payoff is similar to that predicted by the equilibrium. Specifically, the equilibrium ($p=0.579$; $q=0.421$) predicts a mean payoff of 0.9 for a Co member and 0.36 for an Op member per round, while the actual payoffs are 0.83 and 0.40, for Co and Op members, respectively . Given the inherent asymmetry of the *before* game, the members of group Co earned more than twice as much as members of group Op, even though their contribution rates were only slightly higher.

4. Discussion

In this paper we focused on an asymmetric intergroup competition where a tie favors one prespecified group over the other, and asked a question that seems central to the understanding of such situations: The members of which group are more likely to put costly effort into the competition? The example we had in mind is a parliamentary committee, where the opposition typically needs a strict majority to win a vote, whereas for the coalition a tie is sufficient to maintain the status quo.

The intergroup competition was modeled as a participation game, assuming that the group's payoff is a public good and that individual contribution or participation is voluntary and costly. The theoretical analysis of the asymmetric participation game yielded two qualitatively different predictions, one in which members of the coalition are slightly more likely to contribute than members of the opposition, and another in which members of the opposition are much more likely to contribute than members of the coalition. The experimental results clearly support the first prediction. Members of the "coalition" contributed on average on 58% of the rounds as compared with a contribution rate of 51% by members of the "opposition".

[7] A very slow learning process in the symmetric participation game is predicted by Roth & Erev's (1995) learning model. An experiment by Bornstein, Erev, & Goren (1994) corroborates this prediction.

Consequently, a “coalition” member earned more than twice as much as an “opposition” member.

As a baseline for evaluating our results, we compared the asymmetric game to a symmetric one. Based on the theoretical analysis of the two games, we predicted that overall contribution rates would be higher in the symmetric game than in the asymmetric one. The results show that although players contributed much less in the symmetric game than theoretically prescribed, they did contribute somewhat more than players in the asymmetric game.

The participation games operationalized in the present study model quite closely voting in small groups such as parliamentary committees and boards of directors. However, the games can be used, albeit more loosely, to model a larger class of intergroup conflicts and competitions (e.g., wars, soccer games) where a tie or a stalemate, with neither side clearly winning nor losing the competition, is a potential outcome. In some of these competitions, as in our asymmetric game, the utility of a tie may be different for each of the competing sides (Snidal, 1986). One group may only aspire to maintain the status quo and therefore may value a tie as if it were a win, whereas a tie and the ensuing status quo may be valued more like a loss by the other group. To the extent that our experimental results can be generalized to such situations, it seems that less voluntary effort is to be expected from members of the group that must win than from members of the group that need only not lose.

References

Bornstein, G. (1992) "The free rider problem in intergroup conflicts over step-level and continuous public goods," *Journal of Personality and Social Psychology*, 62, 597-606.

Bornstein, G. (2003) "Intergroup conflict: Individual, group, and collective interests," *Personality and Social Psychology Review, 7,* 129-145.

Bornstein, G. & Rapoport, A. (1988) "Intergroup competition for the provision of step-level public goods: Effects of preplay communication," *European Journal of Social Psychology*, 18, 125-142.

Bornstein, G., Erev, I., and Goren, H. (1994) "The effect of repeated play in the IPG and IPD team games," *Journal of Conflict Resolution*, 38, 690-707.

Harley, C. B. (1982) " Learning the evolutionarily stable strategy," *Journal of Theoretical Biology*, 89, 611-633.

Palfrey, T. R. and Rosenthal, H. (1983) "A strategic calculus of voting," *Public Choice*, 41, 7-53.

Rapoport, A. & Bornstein, G. (1987) "Intergroup competition for the provision of binary public goods," *Psychological Review*, 94, 291-299

Roth, A. E., and Erev, I. (1995) "Learning in extensive-form games: Experimental data and simple dynamic models in the intermediate term," *Games and Economic Behavior*, 8, 164-212.

Schram A.J.H.C. and Sonnemans, J. (1996a) "Voter turnout and the role of groups: Participation game experiments," *International Journal of Game Theory*, 25, 385-406.

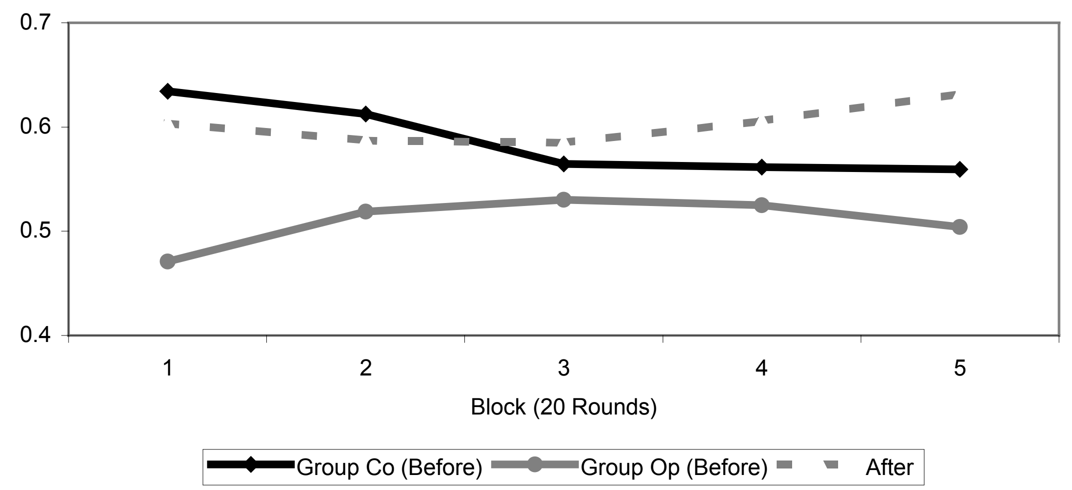
Schram A.J.H.C and Sonnemans, J. (1996b) "Why people vote: Experimental evidence, *Journal of Economic Psychology*, 17, 417-442.

Selten, R. (1991) "Evolution, learning, and economic behavior," *Games and Economic Behavior*, 3, 3-24.

Smith, M. J. (1984) " Game theory and evolution of behavior," *Behavioral and Brain Sciences*, 7, 95-125.

Snidal, D. (1986) "The game theory of international politics," In K. Oye (Ed.), *Cooperation under Anarchy*. Princeton: Princeton University Press.

Figure 1: Mean Contribution Rates



Appendix A: Theoretical analysis

In this section we provide a full analysis of the (within group) symmetric Nash equilibria of the two games in our experiment. We consider all such games parameterized by the payoff parameters e and r. We normalize the payoff units so as to have $r = 1$, making $e/r = e$ to be the only parameter of the payoffs in the game. In our analysis which covers all positive values of e, we consider only equilibrium that is either pure or *symmetric mixed*, that is, equilibrium in which symmetric players use the same (mixed) strategy. In the **after** condition this means that all players use the same mixed strategy while in the **before** condition, all players in the same team use the same mixed strategy, but the two strategies may be different.

Equilibrium analysis of the *after* treatment

Pure equilibrium

A pure strategy profile is (up to permutation of symmetric players) (n,m) where n and m are the number of contributors in the two teams. We shall show that:

- If $e > 1/2$, then the only pure equilibrium is that in which no player contributes.
- If $e < 1/2$, then the only pure equilibrium is that in which all players contribute.
- If $e = 1/2$, then the set of pure equilibria consists of all strategy profiles in which $\mid n-m \mid \leq 1$.

To see this, assume that (n,m) is a Nash equilibrium then,

- For any $0 < e < 1$, $\mid n-m \mid \geq 2$ is impossible since in such a case, any contributor in team with more contributors has a profitable deviation to 'do not contribute'(keeping e and still winning the prize).
- For any $0 < e < 1$; $e \neq 1/2$ it is not possible that $\mid n-m \mid = 1$. First, for any $e > 0$, there can be no contributor in the losing team since such a player could deviate to not contributing (yielding e instead of 0). As to a configuration $n = 1; m = 0$; If $e < 1/2$, a non contributor in the losing team could deviate and contribute (getting 1 with probability $1/2$ which is higher than e). If $e > 1/2$, the contributor could deviate and not contribute (getting $e + 1/2$ which is higher than 1). If $e = 1/2$, such a profile is in fact an equilibrium.
- It remains to check the case $n = m$. It is readily verified that this is equilibrium when $e = 1/2$. Otherwise, $0 < n = m < 3$ is not possible since then, if $e < 1/2$, a non contributor could contribute and get 1 instead of $e + 1/2$. If $e > 1/2$, a contributor could not contribute and get e instead of $1/2$. Finally $n = m = 3$ is an equilibrium if $e < 1/2$ while $n = m = 0$ is an equilibrium if $e > 1/2$.

Symmetric mixed equilibrium

We now look for equilibrium in which each player contributes with probability $p \in (0,1)$; which is the same for all players.

When all other players contribute with probability p, the number Y of contributors in the *out* team (the other team) has a binomial distribution $Y \sim B(3,p)$ and the number X of contributors in the *in* team (among the other two players) has a binomial distribution $X \sim B(2,p)$. In this situation, consider the *additional* expected prize due to the player's contribution (that is, her expected prize if she contributes minus her expected prize when she does not contribute). The contribution of the player affects the prize in two events: When $X = Y$ (in which case, by contributing, the player will win the prize with certainty instead of probability $1/2$) and when $X+1 = Y$ (in which case, by contributing, the player will win the prize with probability $1/2$ instead of losing). Thus her own contribution will increase her expected prize by ($P(A)$ stands for the probability of event A):

$$f(p) := \frac{1}{2}\left[P(X=Y) + P(X+1=Y)\right]$$

The equilibrium condition is that the player is indifferent between contributing or not (since she is using both options with positive probability) that is: $f(p) = e$. Now, by straightforward computation we have:

$$\begin{aligned}
f(p) &= \tfrac{1}{2}\left[P(X=Y) + P(X+1=Y)\right] \\
&= \tfrac{1}{2}\left[(1-p)^5 + 3p(1+p)(1-p)^3 + 3p^3(2-p)(1-p) + p^5\right] \\
&= \tfrac{1}{2} - 2z + 2z^2 \quad \text{Where } \; z = z(p) = p(1-p)
\end{aligned}$$

From this it follows that:

- $f(p)$ is symmetric about $p = 1/2$ that is, $f(p) = f(1-p)$. This is because f is a function of $z(p)$ which has this property, $z(1-p) = z(p)$.
- Since $0 \le z(p) \le 1/4$, and since f is a strictly decreasing function of z, it follows that
$$f(p) \le \frac{1}{2} - 2\frac{1}{4} + 2\left(\frac{1}{4}\right)^2 = \frac{5}{16}$$
- consequently, the equation $f(p) = e$ has no solution if $e < 5/16$, has one solution namely, $p = 1/2$ if $e = 5/16$, and has one z-solution if $e > 5/16$ corresponding to two p-solutions symmetric about $1/2$.

We conclude that,

- If $0 < e < 5/16$, there is no symmetric mixed equilibrium.
- If $e = 5/16$, there is one symmetric mixed equilibrium which is $p = 1/2$.
- If $5/16 < e \le 1/2$, there are exactly two symmetric mixed equilibria p and q with $p+q = 1$.

- If $e > 1/2$, there is no symmetric, strictly mixed equilibrium. (Recall that in this case there is a unique symmetric pure equilibrium in which no player contributes).

pure-mixed equilibrium

We next look for pure-mixed equilibria namely equilibrium points in which all players in one team use the same pure strategy while all players of the other team use the same mixed strategy p. We denote such strategy profiles by $(1,p)$ and $(0,p)$ (corresponding to whether the pure strategy is contribute or not). To find such equilibrium points we note that if $(1,p)$ is an equilibrium then a player who contributes with probability p has to be indifferent between contributing or not, which implies $1/2p^2 = e$ while a player contributing with certainty has to (weakly) prefer contributing, which implies

$$1/2\left[3p^2(1-p)+p^3\right] \geq e$$

These two conditions imply $e \leq 2/9$. Similarly the equilibrium conditions for $(0,p)$ are $1/2(1-p)^2 = e$ and

$$1/2\left[3p(1-p)^2+(1-p)^3\right] \geq e$$

These two conditions cannot be satisfied for $p \geq 0$ and hence there is no such equilibrium. It follows that:

- pure-mixed equilibrium exists only for $0 < e < 2/9$ and then it is of the form $(1,p)$ where $p = \sqrt{2e}$.

In particular, for the parameters of our experiment there is no pure-mixed equilibrium (since $e > 2/9$).

Equilibrium analysis of the *before* treatment

We shall denote the two teams by I (for 'incumbent', the team team that wins in case of tie), and E (for 'entrant', the team that loses in case of tie).

Pure equilibrium

A pure strategy profile is (n,m) where n is the number of contributors in team I and m is the number of contributors in team E. Assume that (n,m) is a Nash equilibrium then,

- It must be that $\mid n-m \mid \leq 1$ since otherwise any of the contributors in the winning team could profit by not contributing (saving e while his team is still winning).

- It cannot be the case that $n = m+1$ since again, any contributor in I would have a profitable deviation (namely, by not contributing, she saves e while the team I still wins).

- It cannot be the case that $m = n+1$ since then, any non contributor in I would have a profitable deviation (namely, by contributing, the I team will win instead of losing and her payoff will be 1 instead of e).
- Thus, if there is an equilibrium it must be the case that $n = m$. Clearly $n = m = 0$ is not an equilibrium since then contribution would be a profitable deviation for any player in E. But $n = m \geq 1$ cannot be an equilibrium since not contributing is then a profitable deviation for any contributor in E.

We conclude that, for any $e \in (0,1)$, *there is no pure Nash equilibrium*. This does not exclude the possibility of equilibrium in which all members of one team use pure strategy while the members of the other team use mixed strategies. In fact it follows from the results of the following section that for $e = 4/9$ the following pure-mixed strategy profiles are equilibria:

- All members of I do not contribute while each member of E contributes with probability $1/3$. (Expected payoffs for the players in the two teams are $(2/3, 1)$.)
- All members of E contribute while each member of I contributes with probability $2/3$. (Expected payoffs for the players in the two teams are $(4/9, 19/27)$.)

Symmetric mixed equilibrium

A symmetric (strictly) mixed strategy profile is a pair (p,q); with $0 < p < 1$ and $0 < q < 1$: Each of the players in team I contributes with probability p while each of the players in team E contributes with probability q.

For (p,q) to be an equilibrium, each player has to be indifferent between contributing or not. As argued before (in the analysis of the **after** game), this means that for each player, the *additional* expected prize due to the player's own contribution (given that all other players are contributing according to (p,q)) must equal to e. If X is the number of contributors among the other two players in her own team, and Y is the number of contributors in the other team, then for a player in I this condition is

$$P(X+1=Y) = e \ \text{ where } \ X \sim B(2,p) \text{ and } Y \sim B(3,q)$$

while for a player in E the condition is

$$P(X=Y) = e \ \text{ where } \ X \sim B(2,q) \text{ and } Y \sim B(3,p)$$

Explicitly, these equations are:

$$\begin{aligned} f(p,q) &:= 3q(1-p)^2(1-q)^2 + 6pq^2(1-p)(1-q) + p^2q^3 = e & (1) \\ g(p,q) &:= (1-p)^3(1-q)^2 + 6pq(1-p)^2(1-q) + 3p^2q^2(1-p) = e & (2) \end{aligned}$$

We first note that the two functions are related to each other by:

$$g(p,q) = f(1-q, 1-p) \tag{3}$$

In particular, for $q = 1-p$, we have $g(p,1-p) = f(p,1-p)$ and the two equations 1 and 2 coincide to yield a single equation:

$$10p^2(1-p)^3 = e \tag{4}$$

It is readily verified that the function $10p^2(1-p)^3$ increases in $(0,2/5)$, decreases in $(2/5,1)$ attaining its maximum at $p = 2/5$ with value $6^3/5^4 = 0.3456$. It follows that:

- If $0 < e < 0.3456$, then there are two equilibrium points of the form $(p,1-p)$ namely, the two solutions of equation (4).
- If $e = 0.3456$ then there is only one equilibrium of the form $(p,1-p)$ namely $(2/5,3/5)$.
- If $e > 0.3456$ then there is no equilibrium of the form $(p,1-p)$.
- For the parameters of our experiment we obtain the following two equilibrium points (by numerical solution of equation 4 with $e = 0.25$):

$$(p_1, 1-p_1) = (0.237, 0.763) \text{ and } (p_2, 1-p_2) = (0.579, 0.421)$$

The analysis up to this stage is in line with that of Palfrey and Rosenthal (1983). In fact our game is a special case of their voting game with the "status quo rule" (in case of tie) and our equations 1 and 2 are special case of their equations (16) and (17) on page 33 for $M = N = 3$. The solutions of the form $q = 1-p$ were treated as a special case which is analytically manageable as it requires to solve a single equation with one variable rather than two (rather complex) equations with two variables. Our contribution here is in what follows: We proceed now to find *all* symmetric mixed equilibria for all values of e.

Proposition 1 *The set of all symmetric mixed equilibria of the form* $p,q)$ *is given by:*

- *If* $0 < e < 216/625$*, then there are exactly two symmetric mixed equilibria and they are of the form* $(p,1-p)$ *. (The two equilibria correspond to two different values of p. The constant* $216/625$ *is* $6^3/5^4 = 0.3456$*.)*
- *If* $216/625 \leq e < 4/9$*, then there are exactly two symmetric mixed equilibria and they are of the form* $(p(q),q)$*, where the function* $p(q)$ *is defined by:*

$$p(q) = \frac{1-6q+6q^2+q\sqrt{3-8q+6q^2}}{1-8q+10q^2} \tag{5}$$

  The two equilibria correspond to two different values of q.

- *If* $e = 216/625$*, then there is a unique symmetric mixed equilibrium* $(2/5, 3/5)$*. This is in accordance to both previous cases which coincide for this boundary value of e.*

- *If* $4/9 < e < 1$ *then there is no symmetric mixed equilibrium.*

Figure 1. provides a complete description of the quasi-symmetric mixed equilibria for all values of e.

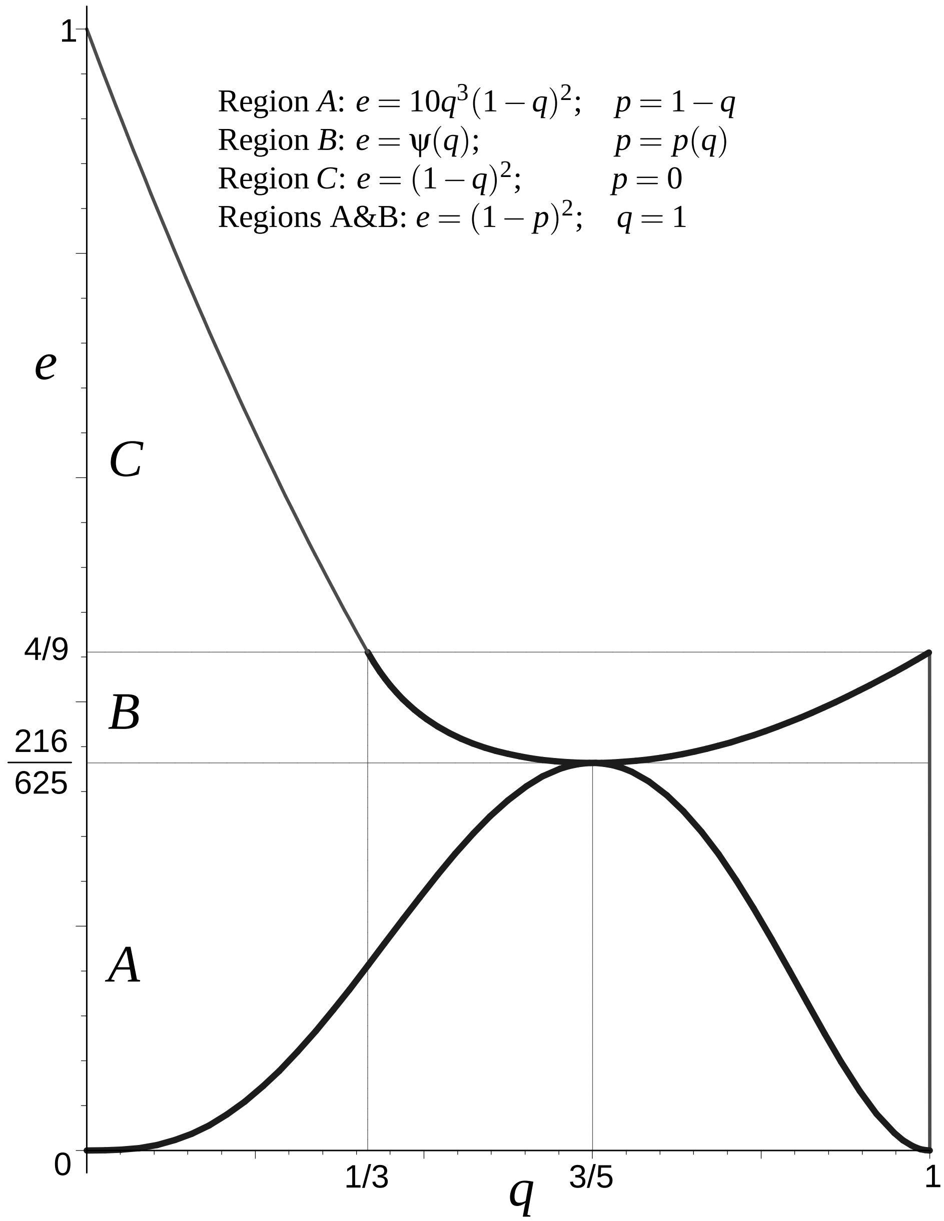


Figure 1. Symmetric equilibria in the **before** game.

Proof of Proposition 1

We start from equations 1 and 2 which, when p and q are strictly between 0 and 1, are necessary and sufficient conditions for (p,q) to be a symmetric mixed equilibrium. A consequence of these two equations is that $f(p,q) - g(p,q) = 0$. As we know that this is satisfied for $p+q=1$, it follows that the left hand side of this equation is divisible by $1-p-q)$ and in fact we find (with the kind help of Mathematica...) that

$$f(p,q) - g(p,q) = (1-p-q)\left(1-4q+3q^2-2p(1-6q+6q^2)+p^2(1-8q+10q^2)\right)$$

As we have already found all solutions for which $1-p-q=0$, all other solutions must solve the equation:

$$a(p,q) := 1-4q+3q^2-2p(1-6q+6q^2)+p^2(1-8q+10q^2) = 0 \tag{6}$$

Solving this as a quadratic function of p and doing some simple, but tedious, algebra we find that the only solutions of equation (6) with both p and q in $[0,1]$ are the pairs $(p(q),q)$ where $1/3 \leq q \leq 1$ and the function $p(q)$ is that given by (5). Note that although the denominator of this function has a zero in the domain, the function is still a smooth (concave) function from $(0,1/3)$ to $(2/3,1)$.

Next, we substitute the value of p given by (5) into equation (1) (or (1)) to obtain the equilibrium condition which becomes (after some algebraic manipulations):

$$\frac{2qs\left[(5-18q+18q^2-4q^3)+(3-8q+6q^2)\sqrt{3-8q+6q^2}\right]}{(1-8q+10q^2)^2} = e \tag{7}$$

The left hand side is a smooth convex function with minimum at $q=3/5$ where $e = 216/625$, and maximum at the boundaries $q=1/3$ and $q=1$ where $e=4/9$ (at both points). Hence for any value of e in the interval $216/625 < e < 4/9$ there are precisely two solutions to the equation (7) which correspond to (the q values of) two mixed equilibrium points (in which the p values are given by (5)). At the minimum point $e=216/625$, there is a unique mixed equilibrium with $q=2/3$ and $p=1/3$. This is the equilibrium at the boundary (the maximum) of the region where the mixed equilibrium is of the form $p+q=1$ (see Figure 1). $\square$

pure-mixed equilibrium

Note that the two boundary points $(0,1/3)$ and $(2/3,1)$ correspond to pure-mixed equilibrium points in which the players in one team use a mixed strategy while the players in the other team use a pure strategy. To find all such equilibria, note that if $(0,q)$ is n equilibrium then equation (2) is still a necessary condition while equation (1) has to be satisfied as an inequality $\leq e$. The equilibrium conditions for $(p,1)$ are equation (2) and equation (1) replaced by an inequality $\geq e$. Similarly for equilibria of the form $(1,q)$ and $(p,0)$. By solving these conditions it is readily verified that all pure-mixed equilibria are given by:

- Equilibrium of the form $(p,1)$ exists only for $0 < e < 4/9$ and then $p = \sqrt{e}$
- Equilibrium of the form $(0,q)$ exists only for $4/9 < e < 1$ and then $q = 1 - \sqrt{e}$ (i.e. $e = (1-q)^2$).
- There is no pure-mixed equilibrium of the form $(p,0)$ or $(1,q)$.

In particular, for the parameters of our experiment, $e = 1/4$, there is a pure-mixed equilibrium which is $(1/2,1)$ namely, all players in the unfavored team contribute while all players in the favored team contribute with probability $1/2$.

Appendix B: Instructions

The after treatment:

You are about to participate in a decision-making experiment. During the experiment you will be asked to make decisions, and so will the other participants. Your own decisions, as well as the decisions of the others, will determine your monetary payoff according to rules that will be explained shortly.

You will be paid in ***cash*** at the end of the experiment exactly according to the rules. Please remain silent throughout the entire experiment and do not communicate in any way with the other participants.

The experiment is computerized. You will make all your decisions by entering the information at the specified locations on the screen. Twelve people are participating in this experiment, which includes 100 decision rounds. At the beginning of each round, the 12 participants will be divided randomly into four groups of three persons each, and each group will be paired with another group. The pairing will be done randomly by the computer. For each new round, the computer will again divide the participants at random into four groups and each group will be paired at random with another group. You will have no way of knowing who belongs to your group and who belongs to the other group.

At the beginning of each round each of you will receive a stake of NIS 0.25 and will have to decide whether to invest your stake or keep it. After all the participants have entered their decisions, the computer will sum up the number of investors in your own group and will compare it with the number of investors in the competing group.

- If the number of contributors in your group is ***larger*** than that in the other group, each member of your group will receive a bonus of NIS 1.

- If the number of contributors in your group is ***smaller*** than that in the other group, each member of your group will receive nothing (0 points).
- If the number of contributors in your group is ***equal*** to that in the other group, one group will be selected at random by the computer, and each member of that group will receive a bonus of NIS 1. Members of the other group will receive nothing (0 points).

At the end of each round you will receive information concerning (a) the total number of contributors in your group; (b) the total number of contributors in the other group; (c) the number of points you earned on that round; and (d) your cumulative earnings up to this point. Then we will move to the next round. Remember that for this new round you will be randomly divided into new groups.

At the end of the experiment the computer will count the total number of points you have earned and we will pay you in cash at a rate of 10 points = NIS 1.

The before treatment:

The instructions for the ***before*** treatment were identical except for the following changes in the payoff rules:

Before each round, one group will be selected at random by the computer to be the winning group in case there is an equal number of contributors in both groups. The identity of this group will be made known to members of both groups.

After reading the instructions, the participants answered a quiz containing three examples. Each example listed the investment decisions of each of the six players, and the participants were asked to fill in the earnings for each player. The experimenter went over the examples and explained the payoff rules until they were fully understood. The examples used in the two treatments were identical.