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One Team Must Win, the Other Need Only Not Lose: An Experimental Study of an Asymmetric Participation Game

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ABSTRACT

We studied asymmetric competition between two (three-person) groups. Each group member received an initial endowment and had to decide whether or not to contribute it. The group with more contributions won the competition and each of its members received a reward. The members of the losing group received nothing. The asymmetry was created by randomly and publicly selecting one group beforehand to be the winning group in the case of a tie. A theoretical analysis of this asymmetric game generates two qualitatively different solutions, one in which members of the group that wins in the case of a tie are somewhat more likely to contribute than members of the group that loses, and another in which members of the group that loses in the case of a tie are much more likely to contribute than members of the group that wins. The experimental results are clearly in line with the first solution. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS intergroup competition; participation game; asymmetric game

1. INTRODUCTION

In this paper we study competition between two groups in which the group whose members contribute more toward their collective group effort wins the competition and receives a reward. The reward is a public good, which is divided equally among the members of the winning group, regardless of whether or not they contributed. Since individual contribution is costly and voluntary, a problem of free-riding is created within each of the competing groups (Bornstein, 2003).

Past research on intergroup conflicts of this ‘‘winner takes all’’ type has focused on the symmetric case where, if the competition is tied, the reward is divided equally between the two groups, or awarded to one of

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the groups at random (Palfrey & Rosenthal, 1983; Rapoport & Bornstein, 1987; Bornstein & Rapoport, 1988; Bornstein, 1992; Bornstein, Erev, & Goren, 1994; Schram & Sonnemans, 1996a, 1996b). In the current investigation we consider an asymmetric competition where, in the case of a tie, one prespecified group receives the reward while the other receives nothing. In other words, to receive the public good one group must win the competition while the other group need only not lose it.

To motivate this line of inquiry consider a parliamentary committee with an equal number of coalition and opposition members. The opposition is trying to pass a motion to which the coalition objects. To pass the motion, the opposition needs a strict majority, whereas for the coalition a tie is sufficient to block the motion and maintain the status quo. The members of both groups decide individually whether to attend the meeting and vote according to their preferences. Attending the meeting is costly (e.g., committee members have to forgo other, more personally beneficial, political activities). The favorable outcome—passing or blocking the motion—is a public good, which is enjoyed equally by all members of the winning group, whether or not they attended the meeting. Such asymmetric competition raises an interesting question: members of which group, the one that wins or the one that loses in the case of a tie, are more likely to attend?

To answer this question we conducted a theoretical and experimental study of an asymmetric participation game. The game involves two competing groups with three members in each group. Each group member receives an initial endowment of e ($e > 0$) and has to decide whether or not to contribute it. Players keep their endowment if they do not contribute, and contributions are not refunded regardless of the outcome of the game.[1] The decisions are made simultaneously, with no opportunity to communicate within or between the groups. The group with more contributions wins the competition and each of its members receives a reward of r ($r > e$). The reward is given to all the members of the winning group regardless of whether or not they contribute (hence the public-good nature of the reward). The members of the losing group are paid nothing. The asymmetry is created by selecting one group beforehand by tossing a coin to be the winning group in the case of a tie. The identity of this group is known to members of both groups.

Section 2 below specifies the Nash equilibria of this participation game. The following sections describe an experiment that studied actual choice behavior in this game.

2. THEORETICAL ANALYSIS

The equilibria of the one-stage asymmetric participation game are computed for the specific payoff parameters used in the experiment, $e = 0.25$, and $r = 1$.[2] The general case for any payoff parameters e and r is reported in Appendix A. In analyzing this game we consider only equilibria that are symmetric within each group, that is, we assume that ex-ante players within each group are undistinguishable, and therefore that all the members of the group that wins in the case of a tie (referred to as group W) contribute with the same probability p, and all the members of the group that loses in the case of a tie (referred to as group L) contribute with a possibly different probability q.

For the set of parameters chosen, the game has three such symmetric equilibria. In one equilibrium, members of group W are more likely to contribute than members of group L ($p = 0.579$; $q = 0.421$). In the other two equilibria members of group L are more likely to contribute than members of group W. In one of these equilibria, the members of W contribute with $p = 0.237$ and the members of L with $q = 0.763$, and in the other, the members of W contribute with a probability of $p = 0.5$, while the members of L contribute with certainty ($q = 1$).

[1]When the individual decision is binary the game can be conceptualized as a participation game where each player decides between participation (which is costly) and non-participation. We shall use the terms contribution and participation interchangeably.
[2]The payoffs are in New Israeli Shekels. Four NIS equaled about 1US$ when the experiment took place.

The expected payoffs associated with each of the three solutions also differ considerably. In the first equilibrium ($p = 0.579$; $q = 0.421$) the expected payoff is NIS 0.90 for a W member and NIS 0.36 for an L member. In the second equilibrium ($p = 0.237$ and $q = 0.763$) the expected payoff is NIS 0.35 for a W member and NIS 0.91 for an L member. Note that in both solutions the sum of p and q is 1.[3] As a consequence the sum of payoffs under these two equilibria is also constant (NIS 1.26). Thus, even though the two equilibria are rather different, neither one is more collectively efficient (pays a higher sum) than the other. The third equilibrium ($p = 0.5$ and $q = 1$) pays NIS 0.25 for a W member and NIS 0.875 for an L member and is thus less efficient than the first two.

This theoretical analysis provides a useful baseline for evaluating the experimental results. It indicates that the asymmetric game has two qualitatively different solutions, one in which the members of group W are somewhat more likely to contribute than members of group L, and another in which the members of group L are *much* more likely to contribute than members of group W. Game theory cannot decide which of these solutions, if any, will be observed. This remains an empirical issue which our experiment may help resolve.

There is one more point that needs to be discussed before describing the experiment. The game-theoretic analysis above applies to the one-stage game, while in our experiment the game was repeated 100 times. In order to avoid the theoretical complications of a repeated game, each experimental session included 12 participants (that is, two unrelated competitions were played on each round) and the participants were re-matched randomly at the beginning of each round. This random-matching protocol (“strangers” design) effectively prevents the players from employing repeated-game strategies of reciprocation, while providing them with an opportunity to learn the structure of the one-stage game and adapt their behavior accordingly.

3. METHOD

3.1. Subjects and design

The participants were 96 undergraduate students at the Hebrew University of Jerusalem. They were recruited by campus advertisements promising monetary reward for participation in a decision-making experiment. The participants were scheduled in 8 cohorts of 12, and were paid contingent on their decisions and the decisions of their counterparts.

3.2. Procedure

The experiment was held in a computerized laboratory. Upon arrival each participant received NIS 10 for showing up and was seated in a separate cubicle facing a personal computer. The participants were given written instructions concerning the rules and payoffs of the game (see Appendix B) and were asked to read these instructions while the experimenter read them aloud. Then the participants were given a quiz to test their understanding. Their answers were checked by the experimenters and, when necessary, explanations were repeated. The participants were also told that to ensure the confidentiality of their decisions they would receive their payment in sealed envelopes and leave the laboratory one at a time with no opportunity to meet the other participants.

Participants played 100 rounds of the game. The number of rounds to be played was made known in advance. At the beginning of *each* round the 12 individuals were randomly divided into three-player groups and each group was paired randomly with another group. This random-matching protocol was carefully

[3] This result holds for all cases where $0 < e/r < 216/625$; see Appendix A.

explained to the participants. One group in each pair was randomly selected to be the winning group in the case of a tie and the identity of this group was made known to members of both groups. The only constraint on this random matching protocol was that each individual was assigned 50 rounds as a member of group W and 50 rounds as a member of group L. The order of these rounds was randomized.

Each player was given an initial endowment of NIS 0.25 at the beginning of each round and had to decide between contributing his or her endowment and keeping it. Following the completion of a round, each player received feedback concerning: (a) the total number of contributors in his or her group; (b) the total number of contributors in the competing group; (c) his or her earnings in this round; and (d) his or her cumulative earnings. Following the last round, the participants were debriefed on the rationale and purpose of the study. They were then paid and dismissed individually without the opportunity to meet the other participants.

4. RESULTS

4.1. Contribution rates

The theoretical analysis uncovered three Nash equilibria, all assuming that the members of the same group contribute with the same probability. These equilibria are qualitatively quite different, since one specifies that the members of group W are slightly more likely to contribute than members of group L, while the other two indicate that members of group L are *much* more likely to contribute than members of group W. The observed behavior clearly corresponds to the first equilibrium. Members of group W contributed at a rate of 0.58 and members of group L at a rate of 0.51. This difference in contribution rates is statistically significant ($t_7 = 2.37$, $p < 0.05$).

Next we examined the distribution of outcomes across the 1600 stage-games played in the experiment. In 42.4% of the games played, group W had more contributors than group L, in 27.5% of the games group L had more contributors than the group W, and in the remaining 30.1% of the games there was an equal number of contributors in both groups (meaning, of course, that group W won the competition and received the reward).[4]

4.2. Dynamics over time

Figure 1 plots the mean contribution rates in groups W and L in each (20-round) block. As can be seen in the figure, in the first 20 rounds of play members of group W contributed with a probability of 0.630, which is somewhat higher than the 0.579 probability prescribed by the equilibrium. However, as the game progressed, the contribution rates in group W decreased. To test whether the decreasing trend in contribution rate is statistically significant, we fitted a regression line for each of the eight independent observations of group W to predict the contribution rate from the block number, and extracted the unstandardized B coefficients. Except for one session, all the B coefficients were negative, indicating a decrease in contribution rates over time. The mean B was -0.01 ($sd = 0.008$), which is significantly different from zero ($t_7 = -3.363$, $p < 0.012$). Members of group L started out with a contribution rate of 0.47 in the first 20-round block, which is slightly higher than the 0.421 contribution rate that the equilibrium predicts, and their contribution level did not change much during the course of the game. The trend in contribution rates in the group L condition is not significantly different from 0 (mean $B = 0.002$, $sd = 0.0017$; $p_7 = 0.35$, n.s.).

[4]The mean payoff per player is similar to that determined by the equilibrium. Specifically, in equilibrium ($p = 0.579$; $q = 0.421$) the mean payoff for a group W member is 0.9 and the mean payoff for a group L member is 0.36 per round, while the actual payoffs are 0.83 and 0.40, for W and L members, respectively. Given the inherent asymmetry of the game, the members of group W earned more than twice as much as the members of group L, even though their contribution rate was only 7% higher.

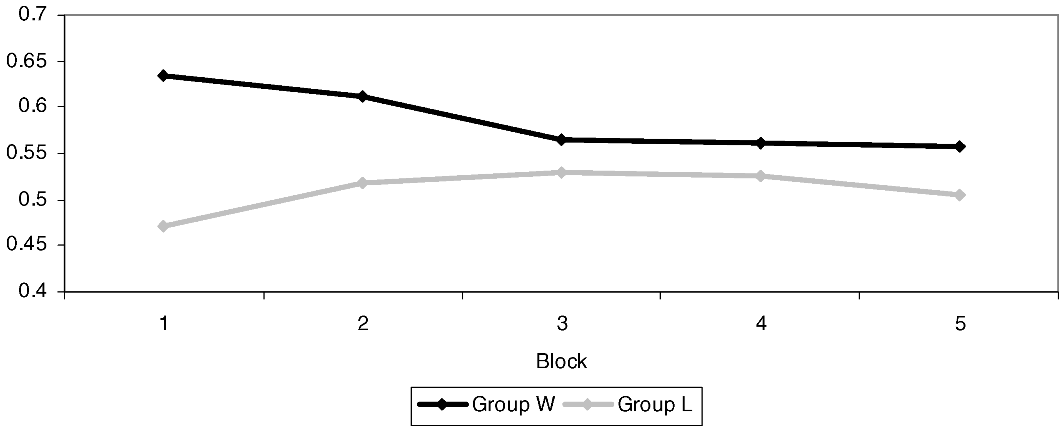

Figure 1. Mean contribution rates

5. DISCUSSION

In this paper we focused on intergroup competition where a tie favors one prespecified group over the other, and asked a question that seems central to the understanding of such asymmetric situations: members of which group are more likely to put costly effort into the competition? The example we had in mind is a parliamentary committee, where the opposition needs a strict majority to win a vote, whereas a tie is sufficient for the coalition to maintain the status quo.

The intergroup competition was modeled as a participation game, assuming that the group's payoff is a public good and that individual contribution or participation is voluntary and costly. The theoretical analysis of the asymmetric participation game yielded two qualitatively different solutions, one in which members of the group that wins in the case of a tie (group W) are slightly more likely to contribute than members of the group that loses (group L), and another in which members of the group that loses in case of a tie (group L) are *much* more likely to contribute than members of the group that wins (group W). The experimental results are clearly in line with the first solution. Members of group W contributed on average in 58% of the rounds as compared with a contribution rate of 51% by members of group L.

It should be noted that the (rather close) resemblance between the observed behavior and a specific theoretical solution holds only for the aggregate *population* level. It does not mean that the *individual* players contributed with the same fixed probability at each period. In fact, it is quite clear that the observed pattern of behavior is not compatible with the standard notion of a mixed strategy. If indeed each player contributed with the same probability p (q) at each period, the average contribution per player (over the 50 decision rounds) would be distributed binomially. Figure 2a compares the theoretical distribution (under the assumption that all players in W contribute with $p = 0.586$—the observed mean contribution rate) with the actual distribution of contributions. Clearly, the two distributions are very different from each other. For example, while theoretically one would expect about half the players to contribute between 50% and 60% of the time, the observed frequency of players in this contribution range is only about 6%. Similarly, while there should be practically no players who contribute between 90% and 100% of the time, or between 0% and 10% of the time, 22% of the players in our experiment almost always contributed, and about 10% of them almost never did. Similar discrepancies between the theoretical and the actual distributions of contribution behavior were observed in group L, as shown in Figure 2b.[5]

[5]The fact that some individuals contributed much while others contributed little can be explained by individual differences in the utility of behaving altruistically or fulfilling one's social duty (e.g., Van Lange et al., 1997; Fehr & Schmidt, 1999). The positive correlation ($r = 0.44$) between a player's contribution in roles W and L clearly points to the existence of such differences.

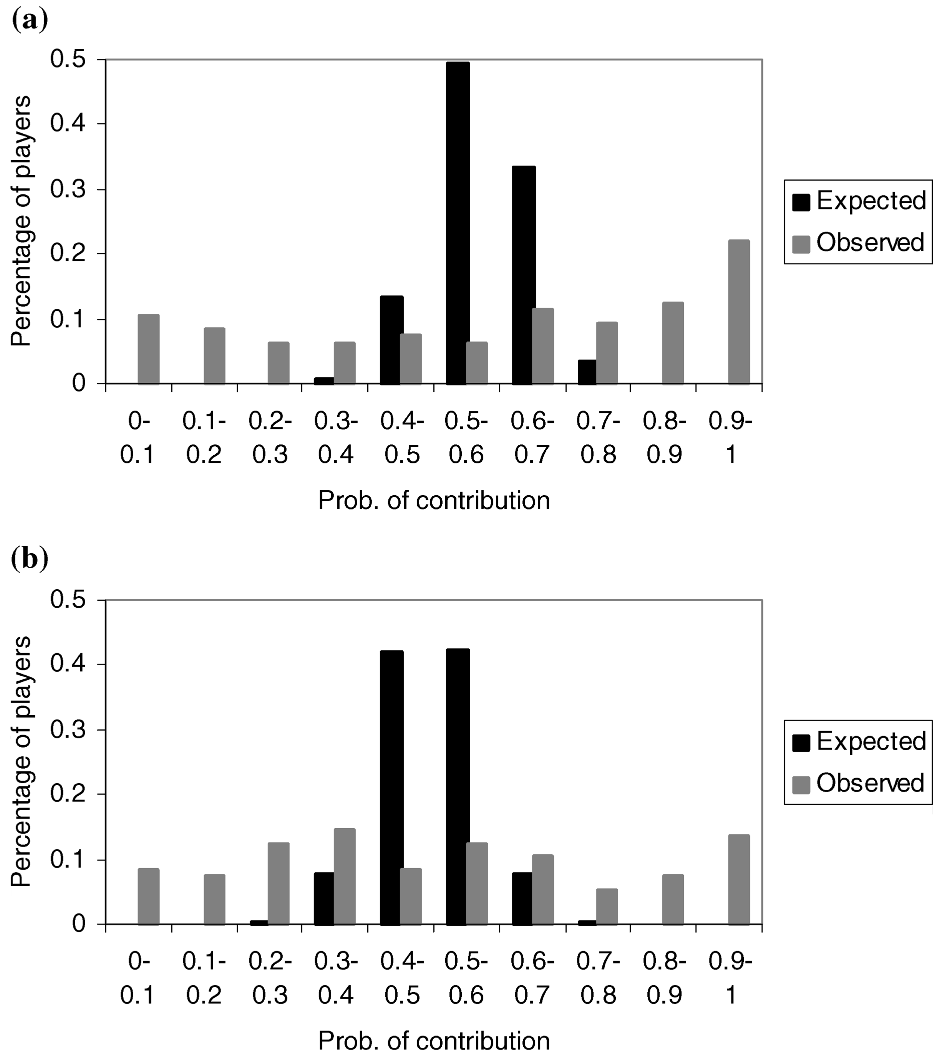

Figure 2. Expected and observed distribution of contribution behavior in groups W (a) and L (b)

The empirical observation that (when, as in the present experiment, players are randomly re-matched with others in the population) a mixed strategy does not characterize individual behavior, but rather the aggregate behavior at the population level, is fairly general and consistent in experimental games (e.g., O'Neill, 1987; Brown & Rosenthal, 1990; Rapoport et al., 1998; Camerer, 2003; Nagel & Zamir, 2004; Rapoport et al., 2004). This phenomenon can be explained by the fact that, in a mixed strategy Nash equilibrium, a player is *indifferent* among her possible actions since they all have the same expected payoff. The optimal behavior depends *only* on the (player's beliefs about the) *average* play of the rest of the population; as long as this average is equal to that generated by an individual play of mixed strategies, the player's best reply is the same (namely, each of her actions is a best reply). In other words, the population game may have many asymmetric Nash equilibria, which will look like a symmetric mixed-strategy equilibrium.

While game theory provides the equilibrium solutions for the game, it does not explain why a particular equilibrium was found to be more salient than the others (Mehta, Starmer, & Sugden, 1994). A possible explanation of this finding is suggested by an earlier experiment on a different kind of asymmetry in participation games (Rapoport & Bornstein, 1989). The experiment studied a ''winner takes all'' competition between a five-person group and a three-person group (with equal endowments to all players and a random tie-breaking rule). The game was played *once*, either with or without an opportunity for pre-play communication within each group. It was found that when group members could communicate they almost invariably decided on the (pure) strategy that maximizes the group's security level and, even though the group's decision was not binding, it was followed by practically all individuals. The maximin (maximizing minimal gain) strategy for the larger group is to designate four contributors (thereby ensuring a payoff of r for each group member), and this strategy was implemented in 87% of the groups. The maximin strategy for the smaller group is to designate no contributors (thus securing each group member a payoff of e), and it was realized in 67% of the groups.[6] Apparently, when group members could decide on a collective strategy, security considerations had a decisive effect on their decision.

But, even when communication was not allowed, the group's maximin strategy seemed to have a substantial effect on individual behavior; 25% of the large groups managed to have exactly four contributors (and 37% of them had three), whereas 37% of the smaller groups had no contributors (and 50% of them had one). As a result, the mean contribution rate in the larger group was 50%, as compared with a mean contribution rate of only 25% in the smaller group.

A similar phenomenon could explain the pattern of results in the asymmetric game studied here. The maximin strategy for group W is to have all group members contribute. This strategy guarantees a tie and a reward of r to each member of that group. The maximin strategy of group L is to have no contributors and secure a payoff of e for each member. If we assume that some players attempted to coordinate on these group-level ''playing it safe'' strategies, we can explain why members of group W were considerably more likely to contribute than members of group L early on in the interaction. As the game progressed, however, players in group W gradually decreased their contributions, as they presumably learned that, given the actual contribution rate in group L, their own contribution was often redundant and they could take a free ride. Members of group L seem to have increased their contribution in the hope of occasionally winning the game (although this trend was smaller and did not reach statistical significance). Given the initial play in groups W and L, any simple learning process (e.g., Smith, 1984; Harley, 1981; Selten, 1991; Roth & Erev, 1995) would cause behavior to converge to the nearest equilibrium, which is for members of group W to contribute slightly more than members of group L. As demonstrated by Van Huyck, Cook, and Battalio (1997), social systems tend to converge to the equilibrium which is nearest to the initial play (even if it is incidental). This ''sensitivity to initial conditions'' (Camerer, 2003) might explain why our subjects did not switch to a different equilibrium during the course of the game.

The participation game operationalized in the present study models quite closely voting in small groups such as parliamentary committees and boards of directors.[7] However, the game can be used, albeit more loosely, to model a larger class of intergroup conflicts and competitions (e.g., wars, soccer games) where a tie or a stalemate, with neither side winning or losing the competition, is a potential outcome. In

[6]Clearly, these two strategies are not in equilibrium. Assuming that there will be no contributors in the small group, the large group should designate one. Knowing that, however, the small group should appoint two contributors and win the game. Of course, the large group should suspect that and appoint three, etc. The game between unequal-size groups, like the asymmetric game studied here, has no equilibrium in pure strategies.

[7]In reality such competition would usually involve communication between the participants. Nonetheless, it is common practice in theoretical and experimental economics to investigate strategic situations as non-cooperative games without any form of communication beyond the formal interaction. It is clearly desirable to study the asymmetric participation game with communication as well, but this has to be left to future investigation. Based on Rapoport and Bornstein (1989), we can surmise that if the game is played only once, within-group discussion would lead the two groups to play their respective maximin strategies (namely, group W is predicted to have three contributors and group L to have none).

some of these competitions, as in our asymmetric game, the utility of a tie may be different for each of the competing sides (Snidal, 1986). One group may only aspire to maintain the status quo and therefore may value a tie as if it were a win, whereas a tie and the ensuing status quo may be valued as a loss by the other group. To the extent that our experimental results can be generalized to such situations, it seems that less voluntary effort is to be expected from members of the group that must win than from members of the group that need only not lose.

APPENDIX A: THEORETICAL ANALYSES

We have carried out a full analysis of the (within group) symmetric Nash equilibria of the asymmetric game. We consider all such games parameterized by the payoff parameters e and r. We normalize the payoff units so as to have $r = 1$, making $e/r = e$ the only parameter of the payoffs in the game. In our analysis, which covers all positive values of $e(e < 1)$, we consider only equilibrium that is either pure or *symmetric mixed*, that is, equilibrium in which all players in the same team use the same (pure or mixed) strategy, but the strategies for the two teams may be different.

We denote the two groups by W (the group that wins in the case of a tie) and L (the group that loses in the case of a tie). We first show:

Pure equilibrium

A pure strategy profile is (n, m) where n is the number of contributors in team W and m is the number of contributors in team L. Assume that (n, m) is a Nash equilibrium. Then:

- It must be the case that $|n - m| \leq 1$, since otherwise any of the contributors in the winning group could profit by not contributing (saving e while her group is still winning).
- It cannot be the case that $n = m + 1$ since, again, any contributor in W would have a profitable deviation (namely, by not contributing, she saves e while group W still wins).
- It cannot be the case that $m = n + 1$ since then any non-contributor in W would have a profitable deviation (namely, by contributing, group W will win instead of losing and her payoff will be 1 instead of e).
- Thus, if there is an equilibrium it must be the case that $n = m$. Clearly, $n = m = 0$ is not an equilibrium, since then contribution would be a profitable deviation for any player in L. But $n = m \geq 1$ cannot be an equilibrium since not contributing is then a profitable deviation for any contributor in L.

  We conclude that, for any $e \in (0, 1)$, *there is no pure Nash equilibrium*. This does not exclude the possibility of an equilibrium in which all members of one team use a pure strategy while the members of the other team use mixed strategies. In fact, it follows from the results in the following section that for the parameters used in the experiment ($e = 1/4$) the following is an equilibrium:
- All members of L contribute while each member of W contributes with a probability 1/2 (expected payoffs for the players in groups W and L are 1/4 and 7/8, respectively).

Symmetric mixed equilibria

A symmetric (strictly) mixed strategy profile is a pair (p, q); with $0 < p < 1$ and $0 < q < 1$. The interpretation is that each of the players in group W contributes with probability p while each of the players in group L contributes with probability q (and all contributions are statistically independent).

For (p, q) to be an equilibrium, each player has to be indifferent about contributing or not. This means that for each player the *additional* expected prize due to the player's own contribution, given that all other players are contributing according to (p, q), must be equal to e. If X is the number of contributors among the other two players in her own group, and Y is the number of contributors in the other group, then for a player in group W this condition is:

$$P(X+1=Y)=e \text{ where } X \sim \mathrm{B}(2,p) \text{ and } Y \sim \mathrm{B}(3,q),$$

While for a player in L the condition is:

$$P(X=Y)=e \text{ where } X \sim \mathrm{B}(3,p) \text{ and } Y \sim \mathrm{B}(2,q).$$

Explicitly, these equations are:

$$f(p,q) := 3q(1-p)^2(1-q)^2 + 6pq^2(1-p)(1-q) + p^2q^3 = e \tag{1}$$

$$g(p,q) := (1-p)^3(1-q)^2 + 6pq(1-p)^2(1-q) + 3p^2q^2(1-p) = e \tag{2}$$

We first note that the two functions are related to each other by:

$$g(p,q) = f(1-q, 1-p) \tag{3}$$

In particular, for $q = 1-p$ we have $g(p, 1-p) = f(p, 1-p)$ and the two equations (1) and (2) coincide to yield a single equation:

$$10p^2(1-p)^3 = e \tag{4}$$

It is readily verified that the function $10p^2(1-p)^3$ increases in (0, 2/5) and decreases in (2/5, 1), attaining its maximum at $p = 2/5$ with value $6^3/5^4 = 0.3456$. It follows that:

- If $0 < e < 0.3456$, then there are two equilibrium points of the form $(p, 1-p)$, namely, the two solutions of equation (4).
- If $e = 0.3456$, then there is only one equilibrium of the form $(p, 1-p)$ namely (2/5, 3/5).
- If $e > 0.3456$, then there is no equilibrium of the form $(p, 1-p)$.
- For the parameters of our experiment we obtain the following two equilibrium points (by numerical solution of equation (4) with $e = 0.25$):

$$(p_1, 1-p_1) = (0.237, 0.763) \text{ and } (p_2, 1-p_2) = (0.579, 0.421)$$

The analysis up to this stage is in line with that of Palfrey and Rosenthal (1983). In fact, our game is a special case of their voting game with the ‘‘status quo rule’’ (in the case of a tie), and our equations (1) and (2) are special cases of their equations (16) and (17) for $M = N = 3$. The solutions of the form $q = 1-p$ were treated as a special case which is analytically manageable as it requires solving a single equation with one variable rather than two. Our contribution here is in what follows: we proceed to find *all* symmetric mixed equilibria for all values of e.

Proposition 1

The set of all symmetric mixed equilibria of the form (p, q) is given by:

- *If* $0 < e < \frac{6^3}{5^4} \left(= \frac{216}{625} = 0.3456\right)$, *then there are exactly two symmetric mixed equilibria and they are of the form* $(p, 1-p)$. *The two equilibria correspond to two different values of p, which are two solutions (the only two in the interval [0,1]) of the equation:*

$$10p^2(1-p)^3 = e$$

- *If* $\frac{6^3}{5^4} \leq e < \frac{4}{9}$, *then there are exactly two symmetric mixed equilibria and they are of the form* $(p(q), q)$, *where the function p(q) is defined by:*

$$p(q) = \frac{1 - 6q + 6q^2 + q\sqrt{3 - 8q + 6q^2}}{1 - 8q + 10q^2} \tag{5}$$

and the value of q is a solution of the following equation:

$$\frac{2q^3[(5 - 18q + 18q^2 - 4q^3) + (3 - 8q + 6q^2)\sqrt{3 - 8q + 6q^2}]}{(1 - 8q + 10q^2)^2} = e$$

which has precisely two solutions in the interval [0,1].

- *If* $e = \frac{6^3}{5^4}$ *then there is a unique symmetric mixed equilibrium (2/5, 3/5). This is in accordance to both previous cases, which coincide for this boundary value of e.*
- *If* $4/9 < e < 1$ *then there is no symmetric mixed equilibria.*

Figure 3 provides a complete description of the quasi-symmetric mixed equilibria for all values of e.

Proof of proposition 1

We start from equations (1) and (2) which, when p and q are strictly between 0 and 1, are necessary and sufficient conditions for (p, q) to be a symmetric mixed equilibrium. A consequence of these two equations is that $f(p,q) - g(p,q) = 0$. As we know that this is satisfied for $p + q = 1$, it follows that the left-hand side of the equation is divisible by $1 - p - q$ and in fact we find that:

$$f(p,q) - g(p,q) = (1 - p - q)(1 - 4q + 3q^2 - 2p(1 - 6q + 6q^2) + p^2(1 - 8q + 10q^2))$$

As we have already found all the solutions for which $1 - p - q = 0$, all other solutions must solve the equation:

$$a(p,q) := 1 - 4q + 3q^2 - 2p(1 - 6q + 6q^2) + p^2(1 - 8q + 10q^2) = 0 \tag{6}$$

Solving this as a quadratic function of p, and doing some simple but tedious algebra, we find that the only solutions for equation (6) with both p and q in [0,1] are the pairs $(p(q), q)$ where $1/3 \leq q \leq 1$ and the function $p(q)$ is that given by (5). Note that although the denominator of this function is zero in the domain, the function is still a smooth (concave) function from (0, 1/3) to (2/3, 1).

Next, we substitute the value of p given by (5) into equation (1) to obtain the equilibrium condition which becomes (after some algebraic manipulations):

$$\frac{2q^3[(5 - 18q + 18q^2 - 4q^3) + (3 - 8q + 6q^2)\sqrt{3 - 8q + 6q^2}]}{(1 - 8q + 10q^2)^2} = e \tag{7}$$

The left hand side is a smooth convex function with minimum at $q = 3/5$ where $e = \frac{6^3}{5^4}$ (at both points). Hence for any value of e in the interval $\frac{6^3}{5^4} \leq e < \frac{4}{9}$ there are precisely two solutions to the equation (7) which correspond to (the q values of) two mixed equilibrium points, in which the p values are given by (5). At the minimum point $e = \frac{6^3}{5^4}$ there is a unique mixed equilibrium with $q = 2/3$ and $p = 1/3$. This is the equilibrium at the boundary (the maximum) of the region where the mixed equilibrium is of the form $p + q = 1$ (see Figure 1).

Pure mixed equilibrium

Note that the two boundary points (0, 1/3) and (2/3, 1) correspond to pure-mixed equilibrium points in which the players in one group use a mixed strategy when the players in the other group use a pure strategy. To find

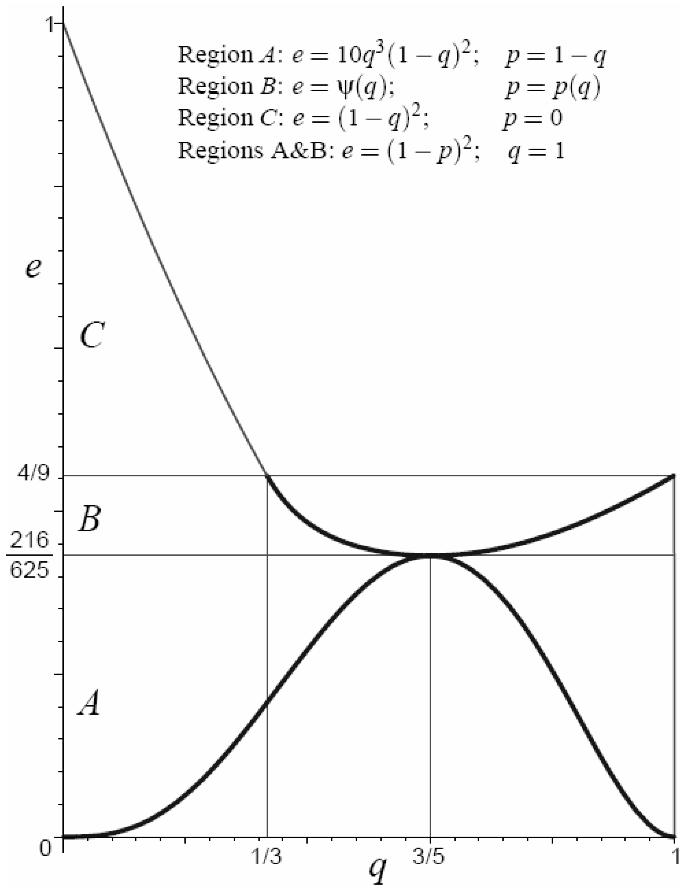

Figure 3. Symmetric equilibria in the game

all such equilibria, note that if (0, q) is an equilibrium then equation (2) is still a necessary condition while equation (1) has to be satisfied as an inequality $\leq e$. The equilibrium conditions for (p, 1) are equation (2) and equation (1) replaced by an inequality $\leq e$. Similarly for equilibria of the form (1, q) and (p, 0). When these conditions are solved it is readily verified that all pure-mixed equilibria are given by:

- Equilibrium of the form (p, 1) exists only for $0 < e < 4/9$ and then $p = \sqrt{e}$
- Equilibrium of the form (0, q) exists only for $4/9 < e < 1$ and then $q = 1 - \sqrt{e}$ (i.e., $e = (1-q)^2$).
- There is no pure-mixed equilibrium of the form (p, 0) or (1, q).

In particular, for the parameters of our experiment, $e = 1/4$, there is a pure-mixed equilibrium which is (1/2, 1) namely, all players in group L contribute with probability 1 while all players in group W contribute with probability 1/2.

APPENDIX B: INSTRUCTIONS

You are about to participate in a decision-making experiment. During the experiment you will be asked to make decisions, and so will the other participants. Your own decisions, as well as these of the others, will determine your monetary payoff according to rules that will be explained shortly.

You will be paid in *cash* at the end of the experiment exactly according to the rules. Please remain silent throughout the entire experiment and do not communicate in any way with the other participants.

The experiment is computerized. You will make all your decisions by entering the information at the specified locations on the screen. Twelve people are participating in this experiment, which includes 100 decision rounds. At the beginning of each round, the 12 participants will be divided randomly into four groups of three persons each, and each group will be paired with another group. The pairing will be done randomly by the computer. For each new round, the computer will again divide the participants at random into four groups and each group will be paired at random with another group. You will have no way of knowing who belongs to your group and who belongs to the other group.

Before each round, one group will be selected at random by the computer to be the winning group in case there is an equal number of contributors in both groups. The identity of this group will be made known to the members of both groups.

At the beginning a round each of you will receive a stake of NIS 0.25 and will have to decide whether to invest your stake or keep it. After all the participants have entered their decisions, the computer will sum up the number of investors in your own group and will compare it with the number of investors in the competing group.

- If the number of contributors in your group is *larger* than that in the other group, each member of your group will receive a bonus of NIS 1.
- If the number of contributors in your group is *smaller* than that in the other group, each member of your group will receive nothing (0 points).
- If the number of contributors in your group is *equal* to that in the other group, each member of your (other) group will receive a bonus of NIS 1. Members of the other (your) group will receive nothing (0 points).

At the end of each round you will receive information concerning: (a) the total number of contributors in your group; (b) the total number of contributors in the other group; (c) the number of points you earned on that round; and (d) your cumulative earnings up to that point. Then we will move to the next round. Remember that for this new round you will be randomly divided into new groups. At the end of the experiment the computer will count the total number of points you have earned and we will pay you in cash at a rate of 10 points = NIS 1.

After reading the instructions, the participants answered a quiz containing three examples. Each example listed the investment decisions of each of the six players, and the participants were asked to fill in the earnings for each player. The experimenter went over the examples and explained the payoff rules until they were fully understood.

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