

# Lower Rank Approximation of Matrices by Least Squares with any Choice of Weights

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## **Abstract**

Reduced rank approximation of matrices has hitherto been possible only by unweighted least squares. This paper presents iterative techniques for obtaining such approximations when weights are introduced. The techniques involve criss-cross regressions with careful initialization. Possible applications of the approximation are in modeling, biplotting, contingency table analysis, fitting of missing values, checking outliers, etc.

Keywords: REDUCED RANK APPROXIMATION  
LEAST SQUARES  
CRISS-CROSS REGRESSION  
HOUSEHOLDER-YOUNG THEOREM  
BIPLOT  
CONTINGENCY TABLE  
OUTLIERS