Common belief and common knowledge

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Abstract

> But game theory as we presently know it can not proceed without the fulcrum of common knowledge. Robert Wilson (1985)

In the universal belief space [Mertens and Zamir (1985)] which incorporated all situations of incomplete information concerning a state space S, we de ne a `knowledge operator' in terms of beliefs. From this operator we derive (in the usual way) the concept of common knowledge and the result is: An event E is common knowledge if and only if it is a belief subspace. Recalling that any game model, with complete or incomplete information, is a belief subspace, this result may be regarded as a considerable weakening of the *common knowledge assumption* that is: If we adopt the universal belief space as a general framework model for incomplete information games, then the statement `the game (i.e. the belief subspace) is Common Knowledge' is a formal provable statement *within* the model. Since a belief subspace may or may not be consistent (in Harsanyi's sense), it follows that with this de nition, and unlike in Aumann's model, players *may agree to disagree*.