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REPEATED GAMES OF INCOMPLETE INFORMATION: THE SYMMETRIC CASE

BY ELON KOHLBERG AND SHMUEL ZAMIR

Harvard University and The Hebrew University

It is shown that in every repeated zero-sum game of incomplete information in which the information to both players is identical, lim v_n and v_∞ exist and are equal.

Repeated games of incomplete information have been of interest for some time (e.g., see [1] and [3]). In this note we consider those repeated two-person zero-sum games of incomplete information in which the information pattern is *symmetric*, that is, the information to both players is exactly the same. The description of such a game is as follows:

The true payoff matrix G^α is one of m possible $r \times s$ matrices $G^1, \cdots, G^m$. The players do not know α but only its probability distribution $p = (p^1, \cdots, p^m)$. The game consists of an infinite sequence of stages. At stage k, players I and II simultaneously choose a row i_k and column j_k, respectively, and the resulting payoff is $G^\alpha(i_k, j_k)$. The players are not informed of the payoff but rather of $H^\alpha(i_k, j_k)$, where $H^1, \cdots, H^m$ are "information matrices." Perfect recall is assumed, that is

$$(1) \qquad i \neq i' \quad \text{or} \quad j \neq j' \Rightarrow H^\alpha(i, j) \neq H^{\alpha'}(i', j') .$$

Let v_n denote the value of the n-times repeated game and v_∞ the value of the infinitely-repeated game (if it exists). Our result is as follows:

THEOREM. *In every repeated zero-sum game of incomplete information with a symmetric information pattern,* v_∞ *and* lim v_n *exist and are equal.*

PROOF. By induction on m (note that this method provides an inductive procedure for computing v_∞).

If v_∞ and lim v_n exist and are equal, we denote their common value by $v(G^1, \cdots, G^m, H^1, \cdots, H^m, p)$.

Let us denote

$$M(\alpha, i, j) = \{\beta \in M : H^\beta(i, j) = H^\alpha(i, j)\} ,$$

where $M = \{1, \cdots, m\}$,

$$p(\alpha, i, j)^\beta = \frac{p^\beta}{\sum_{\gamma \in M(\alpha,i,j)} p^\gamma} \quad \text{if} \quad \beta \in M(\alpha, i, j)$$
$$= 0 \qquad \text{otherwise}$$

and

$$v(\alpha, i, j) = v(\{G^\beta\}_{\beta \in M(\alpha,i,j)}, \{H^\beta\}_{\beta \in M(\alpha,i,j)}, p(\alpha, i, j)) .$$

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If $M(\alpha, i, j)$ contains less than m elements then, by the induction hypothesis, $v(\alpha, i, j)$ is well defined. Our game is now easily seen to be equivalent to the repeated game with payoff matrix (a_{ij}) defined by

$$\begin{aligned} a_{ij} &= \textstyle\sum_{\alpha=1}^{m} p^\alpha G^\alpha(i, j) && \text{if} \quad M(\alpha, i, j) = M \\ &= (\textstyle\sum_{\alpha=1}^{m} p^\alpha v(\alpha, i, j))^* && \text{otherwise;} \end{aligned}$$

here, entries with asterisks are "absorbing" in the sense that, once any of them is reached, all payoffs in future plays must be equal to that same entry (regardless of the players' actions).

Repeated games with "absorbing" payoffs have been studied in [2], where it was shown (Theorem 3.4) that, provided each player knows his opponent's past actions, v_∞ and $\lim v_n$ exist and are equal. In view of (1), the above result may be applied to complete the proof.

REMARK. In [4], Zamir used the same induction to prove the existence of $\lim v_n$ for games with a symmetric information pattern that have 2×2 payoff matrices.

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DEPARTMENT OF ECONOMICS
HARVARD UNIVERSITY
CAMBRIDGE, MASS. 02138

DEPARTMENT OF STATISTICS
THE HEBREW UNIVERSITY
JERUSALEM, ISRAEL