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EXISTENCE OF APPROXIMATE EQUILIBRIA AND CORES

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IT IS WELL KNOWN that for a finite exchange economy, where preferences are not assumed to be convex, there may be no price equilibrium or even the core may be empty. For this reason it was proposed to enlarge the set of price equilibria and the core by introducing the concepts of "approximate equilibrium" and "approximate core."

The existence of approximate equilibria for exchange economies, where preferences are not assumed to be convex, has been investigated by R. Starr [**6**]. He showed that there exists a "quasi-equilibrium," provided the number of participants is large enough and there is a bound on the "degree of non-convexity" [**6**, p. 30, Assumption D]. In this note we shall show the existence of "approximate equilibria" (a stronger concept than the one considered by Starr [**6**, p. 31]) for large economies where the preferences are neither assumed to be convex nor complete. To obtain our result we shall assume that the preferences and the endowments of all participating agents belong to a compact set.

In [**5**] Shapley and Shubik proved that, for a large replica of a given economy with transferable utility, the ε-core is nonempty. We shall generalize this result to large economies without transferable utility by using the concept of ε-core as introduced by Kannai [**2**]. The nonemptiness of the ε-core follows easily from the existence of approximate equilibria and a relationship between the set of approximate equilibria and ε-core.

The existence of ε-core for large economies (with a fixed number of types) can also be deduced from Kannai's Theorem C′ [**2**] in its stronger form (Theorem C″ in [**3**]).

1. THE MODEL AND STATEMENT OF RESULTS

Let T denote an infinite set. For every t in T, there is defined a preference relation $\succ_t$ on the positive orthant Ω of the d-dimensional Euclidian space R^d ($d \geqslant 2$) (i.e., $\succ_t \subset \Omega \times \Omega$; we will write $x \succ_t y$ instead of $(x, y) \in \succ_t$). We assume throughout this paper that for every t in T the relation $\succ_t$ is *irreflexive* (for all x in Ω not $x \succ_t x$), *transitive* (for all x, y, and z in Ω; $x \succ_t y$ and $y \succ_t z$ imply $x \succ_t z$), *open* (the set $\succ_t$ is open in the relative topology of $\Omega \times \Omega$), and *strongly monotonic* (for all x and y in Ω: $x \geqslant y$ and $x \neq y$ imply $x \succ_t y$). (Inequalities between vectors are

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assumed to hold relative to coordinates.) We also define, for every t in T, a vector w_t in Ω, $w_t \neq 0$ (the *initial endowment* of *trader* t).

We denote by S the open simplex

$$\left\{p = (p^1, \ldots, p^d) \in \Omega \mid \sum_{i=1}^{d} p^i = 1 \text{ and } p^i > 0 \text{ for all } i\right\}$$

in R^d. The demand correspondence $\psi : T \times S \to \Omega$ is defined by:

$$\psi(t, p) = \{x \in \Omega \mid px \leqslant pw_t \text{ and } y \succ_t x \text{ implies } py > pw_t\}.$$

It is shown in [4] that ψ is well-defined and, by the monotonicity assumption, $px = pw_t$ holds for all x in $\psi(t, p)$. An *exchange economy* is by definition a finite subset, say E, of T. An *allocation* in the economy E is a collection $\{x_t\}_{t \in E}$ of elements of Ω satisfying $\Sigma_{t \in E}(x_t - w_t) = 0$. In what follows we shall make use of two assumptions on the set T of potential traders:

ASSUMPTION 1—Boundedness of Initial Endowments: *There is a positive real number M such that $|w_t| \leqslant M$ for all t in T* ($|\cdot|$ *being the Euclidean norm in* R^d).

To state our second assumption we need a mathematical concept which reflects the intuitive idea of "similar agents." This concept is a topology on the set of preferences. A precise definition of the topology is postponed to Appendix 1, since it is applied explicitly only in Appendix 2, while through the rest of the paper we need only know that there is such a topology.

ASSUMPTION 2—Compactness of Preferences: *The set* $\{\succ_t\}_{t \in T}$ is compact.

We may note that Assumption 2 is fairly weak since, for instance, the set of all irreflexive, transitive, open, and monotonic[2] preferences is compact in our topology.[3]

Before we state the main theorem we need a few more notations. For a set A we denote by $|A|$ the number of its element. For $a \in R^d$ and $A \subset R^d$, let $\rho(a, A) = \inf_{b \in A} |a - b|$. Further, e will denote the vector $(1, \ldots, 1) \in R^d$.

THEOREM 1: *If T satisfies Assumptions* 1 *and* 2, *then for every $\varepsilon > 0$ and $\delta > 0$, there is an integer $\bar{n}$ such that for every economy E in T with $\Sigma_{t \in E} w_t > |E|\delta e$ and $|E| > \bar{n}$ there is an allocation $\{x_t\}_{t \in E}$ and a price vector $p \in S$ such that:* (i) *for each $t \in E$, $px_t = pw_t$ and $\rho(x_t, \psi(t, p)) < \varepsilon$*; (ii) $|\{t \in E \mid x_t \notin \psi(t, p)\}| < \bar{n}$.

An allocation satisfying (i) and (ii) will be referred to as *$\bar{n}$-bounded ε-equilibrium*. It follows from (i) and (ii) that the aggregate deviation of the traders' bundles from their demands is bounded by a bound independent of the number of traders in the economy, namely $\Sigma_{t \in E}\, \rho(x_t, \psi(t, p)) < \varepsilon\bar{n}$.

[2] A preference relation $\succ$ is said to be monotonic if $x > y$ implies $x \succ y$.

[3] For the proof of this fact which will not be given here we are grateful to B. Grodal.

A *coalition* C is a nonempty subset of E. Given x and y in Ω, we denote by $x \ominus y$ the vector whose jth coordinate is $\max\{0, x^j - y^j\}$ for $1 \leqslant j \leqslant d$.

Let $\{x_t\}$ be an allocation of an economy E, and let ε be a positive number. The allocation $\{x_t\}$ is said to be *ε-blocked* if there is a coalition $C \subset E$ and an allocation $\{y_t\}$ such that: (i) $y_t \succ_t x_t$ for every $t \in C$; (ii) $\Sigma_{t \in C} y_t \leqslant (\Sigma_{t \in C} w_t) \ominus |C|\varepsilon e$. The set of all allocations which are not ε-blocked is called the *ε-core* of E.[4]

For the next result we add the assumption that each trader has a *complete* preference relation on Ω. In our notation this means actually that for all t in T and all x, y, and z in Ω: $x \not\succ_t y$ and $y \not\succ_t z$ implies $x \not\succ_t z$. (Here $x \not\succ_t y$ means: not $x \succ_t y$.) When adding this assumption we have the following theorem.

THEOREM 2: *For every $\varepsilon > 0$ and $\delta > 0$ there is an integer $\bar{n}$ such that every economy E in T with $\Sigma_{t \in E} w_t > |E|\,\delta e$ and $|E| > \bar{n}$ has a nonempty ε-core.*

2. PROOFS

In the proof of Theorem 1 we use the following two lemmas.

LEMMA 1 (Shapley-Folkman): *Let $\{Q_i\}_{i=1}^n$ be a collection of nonempty subsets of R^d. Let x_i be in the convex hull of Q_i for $i = 1, 2, \ldots, n$. Then for each i there is y_i in the convex hull of Q_i such that $\Sigma_{i=1}^n (x_i - y_i) = 0$ and with the possible exception of at most d indices y_i is in Q_i.*

This lemma was used by Starr, and its proof appears in his paper [**6**, Appendix 2, Lemma 2 and Corollary].

LEMMA 2: *Under the conditions of the theorem, there is a number $\eta > 0$ such that for every economy E with $\Sigma_{t \in E} w_t \geqslant |E|\,\delta e$ and every competitive price $p \in S$ of the continuous representation of E it follows that $p \geqslant \eta e$.*

The proof of this lemma is postponed to Appendix 2, but let us explain here the concept of a "continuous representation of a finite economy" (as used by Kannai [**2**]). Given E, we define the following economy with a continuum of traders (Aumann's model). Let $\{I_t\}_{t \in E}$ be a partition of the unit interval such that for all t, I_t is Lebesgue measurable of measure $1/|E|$. For every s in the unit interval we define the initial endowment of s to be w_t and its preference to be $\succ_t$ if s is in I_t. It is obvious that the economy defined in this way fulfills the conditions for existence of competitive equilibrium with incomplete preferences [**4**].

PROOF OF THEOREM 1: Let $(p, f(\cdot))$ be a competitive equilibrium in the continuous representation of an economy E. For t in E we define $x_t = |E| \int_{I_t} f(s)\, ds$. Then x_t is in $\operatorname{conv}(\psi(t, p))$. By Lemma 1 there are y_t in Ω for t in E such that: (a) $\Sigma_{t \in E} (x_t - y_t) = 0$, i.e., $\{y_t\}_{t \in E}$ is an allocation in E, and (b) y_t is in $\psi(t, p)$ with the possible exception of at most d traders t in E, and even then y_t is in the convex hull of $\psi(t, p)$. In particular this implies that $py_t = pw_t$ for all t in E. Using the inequality

[4] This concept has been introduced by Kannai [**2**] under the name of "weak-ε-core."

$p \geqslant \eta e$ obtained from Lemma 2 and Assumption 1 ($|w_t| \leqslant M$ for all t), we conclude that $|y_t - x| \leqslant dM/\eta$ for all x in Ω such that $px = pw_t$. Next, we define z_t as follows: $z_t = y_t$ if $y_t \in \psi(t, p)$ and z_t is an arbitrary point in $\psi(t, p)$ if $y_t \notin \psi(t, p)$. Let $z = \Sigma_{t \in E}(w_t - z_t)$; then $z = \Sigma_{t \in E}(y_t - z_t)$. Hence $|z| \leqslant d^2M/\eta$. Clearly $pz = 0$. The crucial step in our proof is to change the vectors z_t slightly so that the change of each z_t is less than ε and the new vectors form an allocation. To do this we construct d disjoint subsets $\{E_i\}_{i=1}^d$ of E such that

(a) $\quad z_t^i > \delta/d \quad$ for all $\quad t \in E_i$,

(b) $\quad \min\{\varepsilon, \delta/d\} \cdot |E_i| > |z^i|$.

We show later that there is an integer $\bar{n}$ such that this construction is possible provided $|E| > \bar{n}$, and in that case $|\cup_i E_i| < \bar{n}$. The change of z_t is done now as follows: denote

$$D^+ = \{j | 1 \leqslant j \leqslant d; z^j \geqslant 0\}$$

and

$$D^- = \{j | 1 \leqslant j \leqslant d; z^j < 0\}.$$

If $D^- = \varnothing$, then $z = 0$ (since $pz = 0$) and $\{z_t\}$ is already an allocation. Suppose, therefore, that $i \in D^-$; consequently $D^+ \neq \varnothing$. Let $\{a^j\}_{j \in D^+}$ be any positive numbers such that $\Sigma_{j \in D^+} p^j a^j = p^i |z^i|$ and define $a^i = z^i$ and $a^j = 0$ for $j \in D^- \setminus \{i\}$. We define now $\bar{z}_t = z_t + a/|E_i|$ for $t \in E_i$ and $\bar{z}_t = z_t$ for $t \in E \setminus E_i$. By construction of a and by the definition of E_i, $\bar{z}_t \in \Omega$, $a/|E_i| < \varepsilon$, $pa = 0$, and hence $p\bar{z}_t = pw_t$ for all $t \in E$. Furthermore, for $t \in E_i$ we have $\rho(\bar{z}_t, \psi(t, p)) < \varepsilon$ while for $t \in E \setminus E_i$, $\bar{z}_t \in \psi(t, p)$. If we denote $\bar{z} = \Sigma_{t \in E}(w_t - \bar{z}_t)$, then $\{j | 1 \leqslant j \leqslant d; \bar{z}^j < 0\} = |D^-| - 1$. Repeating this procedure a finite number of times (e.g., $|D^-|$) we end up with an allocation as required in the theorem.

Left to be shown is the existence of $\bar{n}$ for which the above mentioned $\{E_i\}_{i=1}^d$ can be constructed. For any i ($1 \leqslant i \leqslant d$), we define $\bar{\bar{E}}_i = \{t \in E | y_t^i > \delta/d\}$; then

$$|E|\delta < \sum_{t \in E} y_t^i \leqslant |\bar{\bar{E}}_i| \, dM/\eta + (|E| - |\bar{\bar{E}}_i|)\delta/d.$$

The first inequality follows from the condition $\Sigma_{t \in E} w_t > |E|\delta e$. The second follows from the definition of $\bar{\bar{E}}_i$ and from the bound on y_t obtained previously. This implies $|\bar{\bar{E}}_i| \geqslant |E|\delta\eta/(2dM)$. Let

$$N = \max\{d^2M/(\eta\varepsilon), d^3M/(\eta\delta)\}$$

and let $\bar{n}$ be any integer greater than $2d^2(N + 1)M/(\delta\eta)$. It is easily verified that $|E| > \bar{n}$ implies $|\bar{\bar{E}}_i| > dN + d$. Now we define $\tilde{E}_i$ to be $\bar{\bar{E}}_i$ if $|\bar{\bar{E}}_i| \leqslant \bar{n}$ and to be any subset of $\bar{\bar{E}}_i$ with cardinality $\bar{n}$ otherwise. Since $(2d/\eta)(M/\delta) \geqslant 1$, we obtain $dN + d \leqslant \tilde{E}_i \leqslant \bar{n}$.

Let

$$\bar{E}_i = \{t \in \tilde{E}_i | z_t^i > \delta/d\}.$$

Since $z_t \neq y_t$ for at most d traders we have $dN \leqslant |\bar{E}_i| \leqslant \bar{n}$. Having the sets $\bar{E}_1, \ldots, \bar{E}_d$, our last step is to choose for each i a subset E_i of $\bar{E}_i$ such that $E_1, \ldots, E_d$ are disjoint, $|E_i| \geqslant N$ and $|U_i E_i| \leqslant \bar{n}$. Such a choice is obviously possible. Finally, condition (a) trivially holds while condition (b) follows from $|E_i| \geqslant N$, the definition of N, and the inequality $|z| < d^2M/\eta$. *Q.E.D.*

To prove Theorem 2, let us prove first the following proposition.

PROPOSITION: *For every* $\varepsilon > 0$ *there is an* $\varepsilon' > 0$ *such that if* $(\{x_t\}, p)$ *is an* ε'-*equilibrium of an economy* E *where* $p \geqslant \eta e$, *then for every* $t \in E$:

$$y_t \succ_t x_t \Rightarrow py_t > pw_t - \varepsilon\eta.$$

PROOF: Assume that the proposition is false. So there is a sequence of (without a loss of generality, disjoint) economies $\{E_n\}_{n=1}^{\infty}$ and for each E_n there is a $(1/n)$-equilibrium $(\{x_t\}_{t\in E_n}, p_n)$, a $t_n \in E_n$, and $y_n \in \Omega$ such that

$$y_n \succ_{t_n} x_{t_n} \qquad \text{and} \qquad p_n y_n \leqslant p_n w_{t_n} - \varepsilon\eta.$$

By compactness and completeness assumptions there are converging subsequences of $\{y_n\}$, $\{x_{t_n}\}$, $\{p_n\}$, $\{\succ_{t_n}\}$, and $\{w_{t_n}\}$ with the limit points $y_0, x_0, p_0, \succ_{t_0}$, and w_0 respectively satisfying: (i) $x_0 \not\succ_{t_0} y_0$ and (ii) $p_0 y_0 \leqslant p_0 w_0 - \varepsilon\eta$. By the definition of $(1/n)$-equilibria, $\rho(x_{t_n}, \psi(t_n, p_n)) < (1/n)$. Now $p_n \geqslant \eta e$ implies that $\psi(t_n, p_n)$ are uniformly bounded. Since ψ is upper hemicontinuous [**1**, Appendix A, IV], it follows that $x_0 \in \psi(t_0, p_0)$ in contradiction to (i) and (ii) and monotonicity of $\succ_{t_0}$.

PROOF OF THEOREM 2: Given $\varepsilon > 0$ and $\delta > 0$, there is an $\eta > 0$ determined by Lemma 2. Let $\varepsilon' > 0$ be determined by the proposition.

According to Theorem 1 there is an integer $\bar{n}$ such that for every economy E with $|E| > \bar{n}$ and $\sum_{t\in E} w_t > |E|\delta e$ there is an ε'-equilibrium $(\{x_t\}, p)$. Furthermore, we know from the proof of Theorem 1 that there is an ε'-equilibrium with p satisfying $p \geqslant \eta e$. We claim that such an ε'-equilibrium of E is in the ε-core of E.

Assume to the contrary that the allocation $\{x_t\}$ is ε-blocked, i.e., there is a coalition C and an allocation $\{y_t\}$ such that

(1) $$y_t \succ_t x_t \quad \text{for every} \quad t \in C,$$

and

(2) $$\sum_{t\in C} y_t \leqslant \left(\sum_{t\in C} w_t\right) \ominus |C|\varepsilon e.$$

By the proposition, $py_t > px_t - \varepsilon\eta$ for all $t \in C$. Hence,

(3) $$p\sum_{t\in C} y_t > p\sum_{t\in C} w_t - |C|\varepsilon\eta.$$

Let

$$\left(\sum_{t\in C} w_t\right) \ominus |C|\varepsilon e = \sum_{t\in C} w_t - (\alpha^1,\ldots,\alpha^d).$$

Without loss of generality we can assume that the ε-blocking coalition C is such that for every $t \in C$ there is a coordinate j_t such that $w_t^{j_t} \geqslant \varepsilon$ (otherwise, $C^t \setminus \{t\}$, ε-blocks $\{x_t\}$ via the same allocation $\{y_t\}$). Hence, $\Sigma_{i=1}^d \alpha^j \geqslant |C|\varepsilon$ and consequently

$$p\sum_{t\in C} y_t \leqslant p\left[\left(\sum_{t\in C} w_t\right) - |C|\varepsilon e\right] \leqslant p\sum_{t\in C} w_t - |C|\varepsilon\eta,$$

in contradiction to (3).

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APPENDIX 1

Topology on the Preferences $\{\succ_t\}_{t\in T}$

For every t in T let

$$F_t = \{(x,y) \in \Omega \times \Omega | x \not\succ_t y\}.$$

The sets F_t are closed in R^{2d} and so we can use the following well-known topology on closed subsets of Euclidian space: the sub-base of the topology consists of the sets

$$\{F_t | F_t \cap K = \varnothing \text{ and } F_t \cap U \neq \varnothing\}$$

where K is a compact set and U is an open set in R^{2d}.

A sequence $(\succ_{t_n})_{n=1,\ldots}$ of preferences converges to a preference $\succ_t$ for this topology if and only if

$$\text{Lim Inf } F_{t_n} = F_t = \text{Lim Sup } F_{t_n},$$

where Lim Inf F_{t_n} is the set of points $(x,y) \in \Omega \times \Omega$ such that every neighborhood of (x,y) intersects all the F_{t_n} with sufficiently large n and Lim Sup F_{t_n} is the set of points $(x,y) \in \Omega \times \Omega$ such that every neighborhood of (x,y) intersects infinitely many F_{t_n}. For a more detailed discussion of this topology and its equivalent formulations, see [**1**, pp. 164–165]. (References to mathematical works on this topology also appear in [**1**].)

APPENDIX 2

Proof of Lemma

Suppose that the lemma is false and there is a sequence of economies $\{E_n\}_{n=1}^{\infty}$ and a sequence of competitive prices (in S) $\{p_n\}_{n=1}^{\infty}$ in their corresponding continuous representations such that, without loss of generality, $p_n \to p$ and $p^1 = 0$. In order to simplify notations, we assume without loss of generality that the economies in $\{E_n\}$ are disjoint. We use the notations of the first paragraph in the proof of the theorem. Thus, for all n, $\{x_t\}_{t\in E_n}$ is an allocation in E_n and x_t is in the convex hull of $\psi(t, p)$ for all t in E_n. Define $\bar{E}_n = \{t \in E_n | \, |x_t| < kM\}$ for some fixed positive number k. Then, by Assumption 1,

$$|E_n|M \geqslant \sum_{t\in E_n} |w_t| \geqslant \left|\sum_{t\in E_n} w_t\right| = \left|\sum_{t\in E_n} x_t\right| \geqslant \left(\sum_{t\in E_n} |x_t|\right)\Big/ d \geqslant (|E_n| - |\bar{E}_n|)kM/d.$$

Hence, $|\bar{E}_n|/|E_n| \geqslant (k - d)/k$.

Next let $D = \{i | 1 \leqslant i \leqslant d,\ p^i > 0\}$. Then there is $\alpha > 0$ such that $p_n^i > \alpha$ for all i in D and all n. Hence, for every i in D and every t in E_n, we have

$$\alpha x_t^i \leqslant p_n x_t \leqslant p_n w_t \leqslant M.$$

The last inequality holds by Assumption 1 on T. So $x_t^i \leqslant M/\alpha$. Let

$$E_n^i = \{t \in E_n | x_t^i > \delta/k\};$$

then

$$|E_n|\delta \leqslant \sum_{t\in E_n} w_t^i = \sum_{t\in E_n} x_t^i \leqslant (|E_n| - |E_n^i|)\delta/k + |E_n^i|M/\alpha$$

where i is in D and δ is from the statement of Lemma 2 (and the theorem). Finally we obtain the inequality

$$|E_n^i|/|E_n| \geqslant (\delta\alpha/M)(k - 1)/k.$$

Now, for any positive numbers δ, α, M, and d there is k such that

$$(\delta\alpha/M)(k - 1)/k + (k - d)/k > 1,$$

i.e., for this k, $\bar{E}_n \cap E_n^i \neq \varnothing$ for all n. We denote by t_n an element of $\bar{E}_n \cap E_n^i$ for $n = 1, 2, \ldots,$.

Let q be a vector in Ω such that $q^i = 0$ for i in D and $q^i(d - |D|) = 1$ for $i \notin D$. Since x_{t_n} is in $\operatorname{conv}\psi(t_n, p_n)$, the intersection

$$\psi(t_n, p_n) \cap \{x \in \Omega | qx \leqslant qx_{t_n}\}$$

is nonempty. Denote by y_n an element of this intersection.

Every y_n, $n = 1, 2, \ldots$, satisfies the inequalities

$$p_n^i y_n^i \leqslant p_n y_n = p_n w_{t_n} \leqslant M$$

and

$$q^i y_n^i \leqslant q y_n \leqslant q x_{t_n} \leqslant kM$$

for $i = 1, \ldots, d$. Hence, $y_n^i \leqslant M/\alpha$ for i in D and $y_n^i \leqslant dkM$ for $i \notin D$. Consequently $|y_n| \leqslant d^2kM/\alpha$ for all n. So the compactness of $\{\succ_{t_n}\}_{n=1}^{\infty}$ and of $\{y_n\}_{n=1}^{\infty}$ imply that there is a subsequence (and to simplify notations, assume that this is the original sequence), a preference relation $\succ_{t_0}$, and y_0 in Ω such that $\succ_{t_n} \to \succ_{t_0}$ and $y_n \to y_0$.

Since $p_n y_n = p_n x_n \geqslant p_n^i x_n^i > \alpha\delta/k$, for every n there is a coordinate $j(n)$ such that $y_n^{j(n)} > \alpha\delta/(dk)$. As $p_n^j \to 0$ for $j \notin D$, it follows that for n large enough $j(n) \in D$. By choosing a subsequence of $\{y_n\}$ (and to simplify notations we denote it also by $\{y_n\}$), there is a coordinate j in D such that $y_n^j > \alpha\delta/(dk) > 0$ for all n.

We denote by e_j the unit vector in R^d whose jth coordinate is 1. For each n let θ_n be defined by

$$p_n(y_n + q - \theta_n y_n^j e_j) = p_n y_n.$$

Clearly $\theta_n \to 0$. Hence, for n large enough, the vectors z_n defined by

$$z_n = y_n + q - \theta_n y_n^j e_j$$

are in Ω, $p_n z_n = p_n y_n = p_n w_{t_n}$, and $z_n \to y_0 + q$. Since y_n is in $\psi(t_n, p_n)$, $z_n \not\succ_{t_n} y_n$. In the limit we obtain $y_0 + q \not\succ_{t_0} y_0$ [**1**, Appendix A, I], a contradiction to monotonicity of $\succ_{t_0}$. *Q.E.D.*

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