Curriculum Vitae of RUTH LAWRENCE
Autumn 1999

BIOGRAPHICAL SKETCH

General

Born: August 2, 1971 Sex: Female Marital status: Married
Nationalty: Israeli/British

Education

1989	D.Phil. University of Oxford (Thesis advisor: M.F. Atiyah)
1989	M.A. University of Oxford
1986	B.A. Class I (Physics) University of Oxford
1985	B.A. Class I with special commendation (Mathematics) University of Oxford

Academic Appointments

1999—	Associate Professor without tenure (Hebrew University)
1997—	Associate Professor with tenure (University of Michigan, Ann Arbor)
1993–1997	Assistant Professor (University of Michigan, Ann Arbor)
1990–1993	Junior Fellow, Society of Fellows (Harvard University)
1989–1990	Lindemann Fellow of the English Speaking Union (held at Harvard)

Institutes visited

June '98	Institut des Hautes Études Sciéntifique, FRANCE
1997–1998	Mathematics Institute, Hebrew University, Jerusalem, ISRAEL
Spring '98	Tel Aviv University, Tel Aviv ISRAEL (semester on Geometry/Topology)
Sept '97	Institut Henri Poincaré, Paris, FRANCE
May-Aug '96	Hebrew University, Jerusalem, ISRAEL
1994–1995	Institut des Hautes Études Sciéntifique, FRANCE
Aug 1993	Mathematics Research Institute, Warwick, ENGLAND
Apr-Aug '92	Mathematics Research Institute, Warwick, ENGLAND
Aug 1991	Research Institute in Mathematical Sciences, Kyoto, JAPAN
Feb-Apr '91	Mathematical Sciences Research Institute, Berkeley
Jun 1990	Max-Planck-Institut für Mathematik, GERMANY
Mar 1989	Institut des Hautes Études Sciéntifique, FRANCE

Feb 1989	Institute for Advanced Study, Princeton (10 days)
Jan 1989	Mathematical Sciences Research Institute, Berkeley

Fellowships and Awards

1999–2002	Joint Principal Investigator, BSF grant 9800119 with D. Bar-Natan, M. Hutchings, V. Jones, L. Rozansky
1999–2002	Guastella Fellowship, held at Hebrew University
1997–2000	Faculty Recognition Award (Michigan)
1997	LS&A Excellence in Education Award, University of Michigan College of Literature, Science & Arts
1996–1998	Principal Investigator, NSF grant DMS-9626544 "Holomorphic invariants of 3-manifolds"
1995–1999	Alfred P. Sloan Foundation Research Fellow
1994–1995	Raymond and Beverley Sackler Fellow Institut des Hautes Études Sciéntifique, FRANCE
1994–1997	Faculty Recognition Award (Michigan)
1990–1993	Principal Investigator, NSF grant DMS-9013738 "Topological Knot Theoretic Connections"
1990–1993	Junior Fellowship of the Society of Fellows (Harvard)
1989–1992	Trinity College Oxford Research Fellowship (deferred/vacated)
1989–1990	Lindemann Research Fellowship (held at Harvard)
1986–1989	Searle Graduate Scholar, St. Hugh's College
1986–1989	SERC Studentship held at Oxford
1983–1986	C.E. Mordan Scholar, St. Hugh's College (Oxford)

Prizes *(all at Oxford)*

1984	College Moderations Prize
1985	College Finals Prize and Hurry Prize
1985	University Junior Mathematics Prize
1986	College Finals Prize

Professional responsibilities

— Co-chair of organising committee for a conference on "Algebraic Aspects of Quantum Field Theory" to be held at MSRI Dec 4-8, 2000.

— Associate Editor of 'Journal for Knot Theory and its Ramifications' 1992+.

— Refereed some 35 papers for various assorted other Journals such as 'Topology', 'Commun. Math. Phys.', 'Inventiones', 'Crelle's Journal', 'J. Funct. Analysis'.

— Reviewed over 20 grant applications for NSF, NSA and NRC.

— Written published reviews of books and papers for *Mathematical Gazette* and *La Gazette des Mathematicien* as well as for *Mathematical Reviews* and *Zentralblatt*, including two featured reviews for *Mathematical Reviews*.

— Organised weekly 'Knot Theory' seminar with visitors 1995+ at UM.

— Undergraduate Counsellor 1993/4, 1995/6, 1996/7, Fall 1999 in UM.

— Faculty Advisor to Undergraduate Mathematical Socity UM Fall 1998 and Fall 1999.

— Colloquium Chair Fall 1998, Fall 1999 in UM.

— Member of Executive Committee of UM Department of Mathematics 1996/7.

— KCP host during Winter '96: contact with groups from public schools in Detroit.

Index theory

After graduating in 1986, I studied as a graduate student at the Mathematical Institute, Oxford under my supervisor, Prof. Sir Michael Atiyah. In my first year as a graduate student I worked on problems connected with index theory, and differential geometry, and my dissertation was on a new 'local' index theorem for twisted Dirac operators.

Topology, the Jones polynomial and CFT

For the next few years I worked on problems related to knot theory, and was, and still am, particularly interested in the numerous connections which exist between mathematics and theoretical physics arising quite naturally, though perhaps surprisingly, when investigating knots and links. This is an area which has expanded very rapidly over the last ten years, since Jones' discovery of a new link invariant and the many reformulations subsequently found, have led to investigations of the underlying connection existing between the different fields involved.

In my Thesis [18], I outlined a new approach to Hecke algebra representations associated with two-row Young diagrams, via a homological construction. Starting from a fibration of configuration spaces of points in punctured complex planes, it is possible to construct a vector bundle on which there is a natural flat connection, the Gauss-Manin connection, whose fibres are obtained as the cohomology of the initial fibration. The holonomy of this connection provides a representation of a braid group. In my Thesis, a suitable twisted abelian local coefficient system was seen to give rise to irreducible representations of the Iwahori-Hecke algebra, $H_n(q)$, associated with 2-row Young diagrams. These constructions were also seen ([14], [15]) to be related to the work of Tsuchiya & Kanie, in which the same representations were obtained via n-point functions in conformal field theory.

I subsequently showed [11] how this leads to a representation of the category of tangles, which specialises, on links, to give a functorial approach to the one-variable Jones polynomial expressed entirely in terms of elementary topology. I also extended the construction to the more general representations of the Iwahori-Hecke algebra occurring in the full (two-variable) Jones polynomial ([6]). The parallel with constructions in conformal field theory also continues in this setting [14], the $\mathfrak{sl}_m$ theory being related to slices $X_L(q, q^{m-1})$ of the 2-variable Jones polynomial in a similar way to that in which the $\mathfrak{sl}_2$ theory is related to the 1-variable Jones polynomial.

Given a Lie algebra $\mathfrak{g}$ and parameter q, there are two main constructions for braid group representations; one is via the monodromy of solutions of the well-known equations of Knizhnik-Zamalodchikov and the other is algebraic, being via R-matrices, obtained from the quantum group $U_q\mathfrak{g}$. Kohno's theorem states that these two representations are isomorphic. The topological construction of my Thesis and succeeding papers provide a bridge between these two representations, the global structure (most naturally obtained via explicit chains and cycles) providing an algebraic representation, while the local structure (most naturally obtained via de Rham cohomology) provides a system of differential equations.

Higher algebra structures

The close connections between the Knizhnik-Zamalodchikov equations, R-matrices and the topological approach, seem to be fundamental in explaining the relationships between conformal field theory, quantum groups and knot theory. They are also linked to many other fields, in many more ways than can be enumerated here. In particular, Drinfel'd's quasi-Hopf algebras provide an algebraic setting for the investigation of braided tensor categories relevant to both sides of Kohno's theorem. Work on integral solutions of K-Z, hypergeometric functions and arrangements of hyperplanes, has been related by many authors to the representation theory of quantum groups and affine Lie algebras. In another direction, the exact solubility of 2-dimensional statistical models is related to quantum groups via the (quantum) Yang-Baxter equation. It is therefore natural to look for higher dimensional analogues of all the elements in this tight web.

I briefly investigated the Manin-Schechtman higher braid groups, $B(n,k)$, which are the natural higher dimensional analogues of the braid groups, within group-like objects, employing configuration spaces of hyperplanes in place of those of points. There is an analogue of the K-Z equation whose monodromy does produce representations of $B(n,k)$ (work of Kohno). I derived an explicit presentation (MSRI preprint) for $B(n,k)$, using cyclic arrangements. Unlike the case of B_n, there is, however, no canonical presentation for $B(n,k)$, despite there being a related strong combinatorial structure, as discovered in work of Manin-Schechtman and others.

Next, I introduced a notion of algebra in higher dimensions ([8], [13]). Ordinary algebra involves products of strings of objects placed in a linear order with an evaluation specified by a bracketting, and a single binary multiplication operation. In the new form of algebra, objects live on oriented simplices, and there are several multiplicative operations, with a suitable notion of associativity. The simplest such type of algebra is what I called a 3-algebra, in which elements live on triangles, and the multiplicative maps $A^{\otimes 3} \longrightarrow A$ and $A^{\otimes 2} \longrightarrow A^{\otimes 2}$, may be geometrically visualised in terms of local moves on triangulations. The analogue of a bialgebra in this context has maps $A^{\otimes j} \longrightarrow A^{\otimes(4-j)}$ $(0 \leq j \leq 4)$, satisfying suitable associativity conditions. Any example of such a structure gives rise to invariants of 3-manifolds. Such examples may be constructed from quantum groups, using quantum $6j$-symbols, and the invariant of 3-manifolds thus obtained is that of Turaev-Viro.

In terms of such higher dimensional algebra structures, I introduced a generalisation of the Yang-Baxter equation as a family of polyhedral commutativity constraints on higher algebra-objects, living on the faces. The r-dimensional version uses the polyhedral types appearing as r-dimensional faces of a large-dimensional permutahedron. For $r = 2$, the system reduces to the usual Yang-Baxter equation. For arbitrary r, one of the equations, namely that associated with a hypercube, is closely related to the sequence of generalisations of YBE, whose first two members are known as the Zamalodchikov and Bazhanov-Strogonov equations. I was able to extend a construction of Carter-Saito which generates solutions to any member of the latter family of equations from solutions of the previous member of that family, to the polyhedral system of equations.

Posets of maximal chains

In 1991, I had already introduced the idea of the existence of a hierarchy of higher algebra structures, which should be applicable to problems in higher dimensions, the first two of which just being ordinary algebras and quantum groups. The whole collection of structures should form a ladder, the passage from one level to the next requiring a 'horizontal' step analogous to passing from a Lie algebra to a universal enveloping algebra, followed by a 'vertical' step of quantisation (deformation). Other authors have also pursued this idea, related to that of *categorification*, although as yet a rigorous mathematical framework for it has not been developed.

It is a general fact that when a geometric structure is encoded algebraically, in moving up one dimension, the new algebraic structure is related to the old one by the, as yet not well-defined, operation of categorification. A close analogy exists with a combinatorial problem in which one desires to place a poset structure on the collection of maximal chains in a given poset. In this sense the Yang-Baxter family of equations is closely related to the permutahedra and I investigated the combinatorial problem of the geometric form of the poset of maximal chains in the symmetric group [4]. This gives rise to some new structures involving what I term 'partitions of partitions'.

Extended topological field theories

I introduced the notion of an *extended topological field theory* (ETFT) [10] with an axiomatic formulation, of the same type as that of Atiyah et al. for TFT's. While ordinary TFT's only consider objects of codimension ≤ 1, ETFT's include structures associated with objects of arbitrary codimension. This requires the use of r-categories and r-vector spaces, which are themselves extensions of the usual notions of a category and a vector space, these being the $r = 1$ cases. An n-dimensional ETFT contains a family of invariants; in particular, a scalar invariant of n-manifolds, a vector invariant of general n-manifolds and other objects associated with lower dimensional manifolds. In terms of triangulated manifolds, it is possible to construct such a theory from a finite quantity of data, which must satisfy a finite number of axioms. The data is most naturally visualised as associated with polyhedra, while the axioms relate to local moves on decompositions into such polyhedra. For 2- and 3-dimensions, the algebraic structures so arising are closely connected to 'usual' algebras and 3-algebras.

There are many questions which remain unanswered at present. For example one wishes to find interesting examples of ETFT's in dimensions >3. An interesting example should be Seiberg-Witten theory in four dimensions, since it is known to satisfy essentially the correct properties to make it into an ETFT; this is a problem in which I continue to be interested. I also believe that such higher-dimensional algebraic structures will prove themselves interesting in their own right as well as useful for studying problems in other fields, such as combinatorics and computational problems.

Current research projects

There are currently in existence a number of different approaches to the Witten-Reshetikhin-Turaev family of quantum invariants of 3-manifolds. For the $\mathfrak{sl}_2$ family of invariants, all current rigorous definitions give a family of invariants $Z_K(M,q)$ indexed by K^{th} roots of unity, q. Work of Murakami and Ohtsuki led to the discovery of certain congruence properties of the coefficients of $Z_K(M,q)$, when viewed as a polynomial in $h = q-1$, for a fixed prime K. Ohtsuki was thereby able to define a formal power series $Z_\infty(M)$ in h which, by its definition, is an invariant of 3-manifolds and is thought to be identical to that obtained from a perturbative expansion of Witten's Feynman integral expression for $Z_K(M,q)$.

In 1995, I was able carry out [9] an explicit calculation of $Z_\infty(M)$ for a special family of simple 3-manifolds M; this was to my knowledge the first such calculation to be performed for manifolds other than Lens spaces. Furthermore, I have been able to show that in these cases and for K prime, the formal power series $Z_\infty(M)$, while having a zero radius of convergence in the complex topology, converges K-adically to $Z_K(M,q)$ when evaluated at a K^{th} root of unity. Similar conclusions for the generalisation from prime K to a prime power were made in [5]. It was also found that, in these cases, $Z_\infty(M)$ could be viewed as the asymptotic expansion of a holomorphic function of $\ln q$, from which the invariants $Z_K(M,q)$ (not just for prime K) could be reconstructed.

In joint work with Lev Rozansky [3], it was shown that these properties also hold for Seifert manifolds, and furthermore that in this case, the Witten–Reshetikhin–Turaev invariant at a K^{th} root of unity may be explicitly written as a sum of terms, each expressible as a holomorphic function of K, the terms being in direct correspondence with equivalence classes of flat connections on the manifold, and with the trivial connection contribution providing the sole contribution to the asymptotic expansion in powers of $h = q-1$, which has become known as the Ohtsuki invariant. This can be summarised in the statement that the *stationary phase expansion for Seifert manifolds is exact.* Witten has suggested that an inherent symmetry in this class of manifolds can be used to prove this by more physical techniques, and this is one of the projects currently underway jointly with Lev Rozansky.

In joint work with Don Zagier [1], it was shown that the invariant for Seifert manifolds possesses an *almost modular structure* in the sense that it can be considered as a $-\frac{1}{2}$ Eichler integral of a theta-function. (The calculations appearing in the paper are for the special case of the Poincaré homology sphere, although the same idea applies more generally.)

I believe that similar properties will hold more generally and in particular that the correct way to view quantum invariants of 3-manifolds is not as a collection of isolated values at roots of unity, but rather as a holomorphic function, $Z_\infty(M)$, much as quantum invariants for links in S^3 are polynomials. I have a number of conjectures concerning the properties of this function and its relation to WRT invariants; see [5].

The construction of the Z_∞ invariant in a manner which allows explicit calculation, its relation to quantum invariants at roots of unity and the investigation of the special properties of these holomorphic functions are the main initial goals of the present project. For invariants related to quantum groups, other than $U_q\mathfrak{sl}_2$, it is expected that this will involve some deep and interesting properties of quantum groups and their representations.

The second goal of the current project is that of understanding, in a combinatorial manner, the coefficients of the formal power series $Z_\infty(M)$ in $h = q - 1$ obtained by an asymptotic expansion around $q = 1$. These coefficients, for the case of $\mathfrak{sl}_2$, are 3-manifold invariants of finite type, in the sense of Garoufalidis and Ohtsuki, the first of which is the Casson invariant. They are known to be all integers for $\mathbf{Z}$ homology spheres and hope to obtain closed geometrically meaningful formulae for them in terms of a combinatorial description of the manifold. The universal Aarhus or LMO invariant allows such a description, although as yet very few concrete calculations have been carried out. One expects formulae comparable with results obtained by perturbative expansion of the path integral formulation of the Witten-Reshetikhin-Turaev invariants, which involve the graph cohomology of the manifold. Some divisibility properties for the first three coefficients in Z_∞ were obtained in [3], while [2] analysed Ohtsuki's original work so as to obtain an explicit combinatorial formula for this series. This formula has a formal representation in a shape similar to the state-sum expression for Z_K, but involving an integration over a continuous colour parameter placed on components of the link, instead of a sum over representation labels. This hints at a strong connection with the representation theory of $SL_q(2, \mathbf{R})$, another area currently under investigation.

In this line there is joint work underway currently with Dror Bar-Natan (on computations of universal manifold invariants), with Lev Rozansky (on Heckman-Duistermaat formulae as applied to Seifert manifolds) and with Don Zagier (looking for almost modularity for invariants of hyperbolic manifolds and connections with the Kashaev conjecture).

One long-term goal of this project is a better 'literal' understanding of those Feynman integrals appearing in the context of topological quantum field theories and perhaps also of Feynman integrals in general. For many mathematicians they are still considered to be in the category of black magic, much as were divergent series until the beginning of this century, when the development of complex analysis brought many techniques which have proved themselves important in fields from p.d.e.s to number theory. In the hands of theoretical physicists, Feynman integrals have proved themselves to be an important tool. It has provided a guiding light to uncover new and interesting mathematics as well as to illuminate old ideas.

PUBLICATIONS

Articles in journals and proceedings volumes

[1] **"Modular forms and quantum invariants of 3-manifolds"**
— *Asian J. Math.* **3** (1999) 93–107.
Special volume dedicated to Sir Michael Atiyah on the occasion of his 70$^{\text{th}}$ birthday.

[2] **"On Ohtsuki's invariants of 3-manifolds"**
— *J. Knot Th. Ramif.* **8** (1999) 1049–1063.

[3] **"Witten-Reshetikhin-Turaev invariants of Seifert manifolds"**
— joint with Lev Rozansky *Commun. Math. Phys.* **205** (1999) 287–314.

[4] **"Yang-Baxter type equations and posets of maximal chains"**
— *J. Comb. Th. A***79** (1997) 68–104.

[5] **"Witten-Reshetikhin-Turaev invariants of 3-manifolds as holomorphic functions"**
— in *'Geometry and Physics'*
Eds. J.E. Andersen, J. Dupont, H. Pedersen, A. Swann
Lecture notes in Pure and Applied Mathematics publ. Marcel Dekker **184** (1996) 363-377

[6] **"Braid group representations associated with $\mathfrak{sl}_m$"**
— *J. Knot Th. Ramif.* **5** (1996) 637–660.

[7] **"An Introduction to Topological Field Theory"**
— *Proc. Symp. Appl. Math.* **51** (1996) 89–128.

[8] **"Algebras and triangle relations"**
— *J. Pure Appl. Alg.* **100** (1995) 43–72.

[9] **"Asymptotic expansions of Witten-Reshetikhin-Turaev invariants for some simple 3–manifolds"**
— *J. Math. Phys.* **36** (1995) 6106–6129.
Invited contribution for the Special Issue on *Quantum geometry and diffeomorphism invariant quantum field theory.*

[10] **"Triangulations, categories and extended topological field theories"**
— in *Quantum Topology*, a collection of papers,
Ed. R. Baadhio and L.H. Kauffman, publ. World Scientific (1993) 191–208.

[11] **"A functorial approach to the one-variable Jones Polynomial"**
— *J. Diff. Geom.* **37** (1993) 689–710.

[12] **"Fluorescent transfer of light in dyed materials"**
— joint paper with S.D. Howison,

SIAM J. Appl. Math. **53** (1993) 447–458.

[13] **“On algebras and triangle relations”**
— in *‘Proc. 2nd. Int. Conf. on Topological and Geometric Methods in Field Theory, Turku, Finland, 26th. May–1st. June, 1991.’*
Ed. J. Mickelsson, O. Pekonen, publ. World Scientific (1992) 429–447.

[14] **“Connections between CFT and Topology via Knot Theory”**
— in *Lecture Notes in Physics* **375** (1991) 245–254.

[15] **“Homological representations of the Hecke algebra”**
— *Commun. Math. Phys.* **135** (1990) 141–191.

[16] **“Topological approach to the Iwahori-Hecke algebra”**
— *Int. J. Mod. Phys. A* **5** (1990) 3213–3219.

[17] **“A universal link invariant”**
— in *‘Proceedings of the IMA conference on Mathematics-Particle Physics Interface, Oxford, England, 12th.–14th. September, 1988.’*
Ed. D.G. Quillen, G.B. Segal, Tsou S.T., publ. Oxford University Press (1990) 151–156.

[18] **“Homology representations of braid groups”**
— Oxford D.Phil. thesis June 1989.

[19] **“Universal Link Invariants using Quantum Groups”**
— in *‘Proceedings of the XVII Int. Conf. on Differential Geometric Methods in Theoretical Physics, Chester, England, 15th.–19th. August, 1988.’*
Ed. A. Solomon, publ. World Scientific (1989) 55–63.

Chapters in books

[20] Appendix to **“Elliptic Curves”**
— in *Graduate Texts in Mathematics No. 111,* Publ. Springer-Verlag (1986).

Book and article reviews

[21] **“Hyper-Kähler geometry and invariants of three-manifolds”**
by L. Rozansky and E. Witten *Selecta Mathematica* **3**(1997) 401–458.
— Specially featured review in *Mathematical Reviews* MR 98m:57041

[22] **“Temperley-Lieb Recoupling Theory and Invariants of 3–Manifolds”**
by L. Kauffman and S. Lins Princeton University Press (1994)
— Review in *La Gazette des Mathematicien* 1995

[23] **“Higher algebraic structures and quantisation”**
by D.S. Freed *Commun. Math. Phys.* **159**(1994) 343–398.

— Specially featured review in *Mathematical Reviews* MR 95c:58034

[24] **"Coxeter graphs and towers of algebras"**
by F.M. Goodman, P. de la Harpe and V.F.R. Jones Publ. Springer-Verlag (1989)
— Review in *The Mathematical Gazette* **75** (1991) 259–260.

SELECTED MAJOR ADDRESSES AND COURSES

Series of three lectures on 'Combinatorial and Algebraic Structures in knot and 3-manifold invariants' — Tel Aviv University special semester on 'Geometry and Topology' (1998).

Invited plenary address (one of four) 'Geometry and algebra: A Unified Voice'
— American Mathematical Society Central Section Meeting (1996).

Invited minicourse on 'Topological QFT and connections with CFT'
— Luminy, Marsailles (1995).

Short course on 'Topological quantum field theories'
— American Mathematical Society Annual Conference (1995).

Series of four lectures on 'Topological Field Theory'
— IHES, France (Oct/Nov 1994).

Invited one hour address at the Inauguration Symposium of the
'Centre for Gravitational Physics and Geometry' at Penn. State U.,
— one of six speakers, two of the others being Roger Penrose and John Wheeler (1993).

Invited talk in special session at AAAS*93
— American Association for the Advancement of Science (1993).

Panel member of discussion group on Mathematics
— 5th Annual Symp. on Frontiers of Science, National Academy of Sciences (1993).

Seminar series of eight lectures at the University of Warwick
— 'Jones polynomials and the topology of configuration spaces' (May/June 1992).

Invited talks at special sessions in American Mathematical Society Meetings
Memphis TN (Mar 97), College Park MD (Apr 97), Jerusalem ISRAEL (May 1995),
Manhattan KS (Mar 1994), San Antonio TX (Jan 1993), Dayton OH (Oct 1992).

Invited lectures at International Conferences
International Workshop on Invariants of Three Manifolds (Calgary, CANADA 1999)
Fifth Amitsur Memorial Symposium (Jerusalem, ISRAEL 1999)
International Conference on Hyperplane Arrangements (Northeastern U., Boston 1999)
International Workshop on Geometry and Topology (Haifa, ISRAEL 1999)
Midwest Topology Conference (Ann Arbor, 1998)
International Workshop on Topology (Tel Aviv, ISRAEL 1998)
Technion 'Geometric Methods in Analysis' (Haifa, ISRAEL 1997)
Institut Henri Poincaré 'Integrales Fonctionnelles' (Paris, FRANCE 1997)

Fields Institute ‘Homotopy geometry and physics’ (Toronto, CANADA 1996)
‘Integrable systems and related subjects’ (Tolaga Bay, NEW ZEALAND 1996)
‘Geometry and Physics’ (Aarhus, DENMARK 1995)
3rd Ann Arbor Combinatorics Conference (1994)
Symp. on ‘CFT, operator algebras and low-dim topology’ (Warwick, ENGLAND 1993)
Quantum deformations of algebras and their representations (ISRAEL 1992)
Int. Symp. on Toplogical and Geom. Meth. in Field Theory (Turku, FINLAND 1991)
Knots 90 (Osaka, JAPAN 1990)
Int. Conf. on Differential Geometric Methods in Theoretical Phys.
—17$^{\text{th}}$(Chester, ENGLAND 1988), 19$^{\text{th}}$(Rapallo, ITALY 1990), 20$^{\text{th}}$(New York 1991)

Mathematics Colloquia

Boston University (1991), Brown University (1993), Columbia (1990),
Cornell (1989), U.C. Davis (1991, 1996), Hebrew University (1998),
Jerusalem College of Technology (1997), Michigan State U. (1996), Ohio State U. (1997),
Osaka City University (1990), Tel Aviv University (1998), Wayne State U. (1996).

A total of **over 150** research lectures at various institutions throughout the world.

PUBLIC LECTURES

Math Camp on ‘Knots and Polyhedra’
— a two-week intensive course for schoolchildren, Ann Arbor (1997).

Public lecture on ‘The Nature of Topology’ in Kyoto (1991).

Japan Association for Mathematical Sciences lecture for Japanese schoolchildren
— ‘Links in mathematics and physics’ (1990).

Japan Association for Mathematical Sciences invited address
— International seminar bringing together scientists from Russia, China, USA and Japan
‘Knots and their connections’ (1990).

Royal Society public lecture ‘When is a knot not a knot?’ (1989).

Special lecture for Schools British Mathematical Association
— ‘Why mathematicians are tied up in knots’ (1988).

Minicourse ‘Some knotty problems’ to school teachers
— British Mathematical Association Annual Conference (1988).

TEACHING

While I was at Oxford, I gave an average of three tutorials a week, every term between Hilary 1986 and Summer 1988, to first year Physics students at St. Hugh's College Oxford.

During my first year at Harvard, I gave a full semester Advanced Graduate Course on Knot Theory, which covered some of the many approaches which have been developed this century including the then new connections with Chern-Simons theory.

Since moving to Michigan, I have taught thirteen regular courses and three reading courses (on knot theory, quantum mechanics and tensors), all of which I have enjoyed giving. Although it is not possible in a traditional class enviroment to give as individual an attention to students' needs as I would like to give, I believe that there are other ways in which this can be remedied. In particular, I believe that a really good way of addressing students' difficulties in a specifically individual way is through written comments on homework and by personal contact with students during office hours. I therefore have never used a grader at UM and have consistently encouraged students to come to my office hours. Consequently, my office hours for undergraduate courses are often fully occupied. Despite the extra time this takes, I feel that the reward of seeing students starting to do mathematics with understanding and enjoyment, makes this effort worthwhile.

I have given three major short courses while I have been based at UM. The first was a series of four lectures delivered at the Institut des Hautes Etudes Scientifiques on *Topological Field Theory*, FRANCE in Fall '94. It gave an introduction to contemporary ideas in TFT and was aimed at graduate students and researchers in other fields of mathematics; the audience ranged from over 80 in the first lecture, to around 40 in the last and consisted of mathematicians, physicists and graduate students in the Paris area.

The second was a lecture as part of a short course delivered at the Annual Meeting of the American Mathematical Society in San Francisco in Jan '95 on *Topological quantum field theories*, to an audience of around 70 US graduate students, along with a number of researchers in the area. The aim was an introduction to the basic notions and the demonstration of connections with geometry, topology, combinatorics and category theory.

The third was a two-lecture Minicourse at Luminy, Marsailles in Aug '95 on *Topological Quantum Field Theory and its connections with Conformal Field Theory*. The audience consisted of some 40 graduate students in mathematics and physics, from France.

In June/July '97, I gave an intensive two week course entitled 'Knots and Polyhedra', to a selected group ('Math Camp') of young area students (ages 13-17). The format of a typical day consisted of a 3 hour intensive period in the morning of lecture and directed work, after which I handed out a list of projects at varying levels to be attacked in the afternoon session. The students each picked projects as they desired (some constructional) according to their level, and worked under the guidance of a graduate student as a facilitator. There were also some video presentations from the Geometry Center in afternoons. The aim was to give a feeling for what mathematics is, its structure, interconnections, ideas and applications, rather than to teach any particular topic. However we encountered many deep concepts from group theory, combinatorics and topology in the tour!

Math 590 (Fall '93) An introduction to point-set topology. *Enrollment: 12*

This is a core graduate course, although some strong undergraduates and engineering graduate students attended.

Math 217 (Winter '94, Fall '95 and Fall '99) Linear Algebra. *Enrollment: 19, 28 and 29.*

This is a core undergraduate course for Mathematics Concentrators and the first place where many of the students have met notions of *rigour* and *proof*. A major emphasis of the course is to get such ideas over to students along with the concept of *abstraction* which enables results from mathematics to be used in applications to almost any field and is central to mathematical thinking. In this respect the subject of the course, linear algebra, is subsiduary.

Despite there being a very large number of elementary 'Linear Algebra' textbooks, I did not find any which approached the subject with what I considered appropriate geometric motivation. Thus during Winter '94, I constructed a wholely new course, complete with quizzes and homework problems. The generation of a complete set of printed course notes during my first experience of teaching the course in '94 enabled my students in the second presentation of this perspective on the course in Fall '95 to use this material in place of a textbook. I believe that on both occasions most of the students left the course with a comparatively solid understanding of the main concepts of linear algebra and, more importantly, gained some understanding of what 'doing' mathematics is all about.

Math 697 (Winter '94) Topics in Topology: Knot Theory *Enrollment: 16*

This 'Topics in Topology' course was devoted to knot theory, a discussion of its origins and classical topological techniques as well as the field of 'quantum topology', which has come into existence over the past decade. The emphasis was on giving an acquaintanceship with the many areas which have been seen to border on knot theory (representation theory, quantum groups, conformal field theory, axiomatic topological field theory, von Neumann algebras, quantum groups) and on demonstrating some of their interconnections.

Math 694 (Fall '95) Topics in Topology: Knot Theory *Enrollment: 6*

As a number of students in this course had attended my previous Knot Theory course, I concentrated more on 3-manifolds than on links, discussing in the second half of this course various constructions of quantum invariants of 3-manifolds and emphasising their relationships and the underlying combinatorial and categorical structures. The approach was almost entirely geometric without algebraic or analytic connections.

Math 316 (Fall '95 and Fall '98) Differential Equations *Enrollment: 17 and 16*

This is a core undergraduate course for Mathematics Concentrators, although a number of Economics majors attended.

Math 115 (Fall '96) Calculus (Freshman) *Enrollment: 29*

This is the first freshman calculus course which I have taught and uses techniques of group learning and team homework with which I was previously unfamiliar, although it does clearly have some advantages for the students.

Math 590 (Fall '96) Geometry and Topology *Enrollment: 17*

This was an upper-level undergraduate introduction to point-set topology was given at an unusually high level due to the qualities of the class.

Math 450 (Winter '97) Advanced Calculus *Enrollment: 21*

This was a thorough course in vector calculus with applications. The students were mainly graduate Engineering students, although some exceptionally bright undergraduates attended.

Math 425 (Fall '98) Introduction to Probability *Enrollment: 26*

This was a course introducing students to basic ideas in probability theory, continuous and discrete distributions, the central limit theorem etc. The students were mixed, many being only needing probability as a tool in their own subjects.

Infi Mitkadem (Spring '99) Differential geometry/elementary functional analysis *Enrollment: around 40 (Hebrew University)*

This course was the fourth of a sequence of calculus courses. Its contents was somewhat non-standard this year, consisting of what was left over from previous courses in the sequence – differential geometry of curves and surfaces, some point-set topology as well as some elementary functional analysis. This course was given in Hebrew.

Math 216 (Fall '99) Differential Equations *Enrollment: 84*

This was a course on differential equations (with some linear algebra), primarily intended for engineers. It lays stress on the methods as opposed to theory, including some numerical methods and five computer labs.

Graduate Students

I have had one student, J. Sink, complete his Ph.D. with me (Spring '99) on "Asymptotic Expansions of Quantum Invariants and a zeta-function of a knot". I also had close contact with three other graduate students at UM, for varying periods, although all longer than a year; however, mainly because of moving back and forth between continents, I moved them onto other advisors last year. I am currently in contact with two students at HU, one of whom hopes to complete his M.Sc. with me this year, and the other hopes to work with me next year.