Syllabus for MATAP 2 – Draft (18/02/09)

Vector analysis (3 lectures)

- Green's thm. / Stokes' thm. / Gauss thm. (some explanation of proof for special regions, gluing regions – ctd from MATAP 1)
- Example of derivation of law of continuity etc.
- Einstein summation convention; ε, δ tensors and relations
- Use in derivation of relations amongst div, curl, grad
- $\nabla^2 \equiv \Delta$ in other coordinate systems
- local / global existence of scalar / vector potential (ctd from MATAP 1)
- decomposition of vector field into transverse ($\nabla \phi$) / longitudinal ($\mathrm{div} = 0$)
- brief discussion of tensors

Recurrence relations (1 class)

- some simple examples / initial conditions
- solution of linear recurrence relations using linear algebra (constant coefficient case in detail)

Differential equations (8 classes)

- general forms of differential equations and their types / initial conditions
- first order ODEs
  - * some methods of solution for special types (variables separable, homogeneous, exact, linear / integrating factor)
  - * special equations & techniques using change of variables (e.g. Bernoulli, Clairaut, $y'' = f(y, y')$)

(3 classes)

- existence and uniqueness theorems
- equivalence of higher order ode to system

DOD →

- theory of linear differential equations / systems
  - * fundamental solution set / Wronskian
  - * solutions of homogeneous / non-homogeneous equations
  - * operator formulation
- higher order linear ODEs
  - * constant coeff. ODE (both $\sum a_r y^{(r)}$, $\sum a_r x^r y^{(r)}$)
  - * methods of finding particular solutions (undetermined coeff. / variation of param)
  - * reduction of order given one solution

(5 classes)

- first order linear systems of ODEs
  - * constant coeff. systems / exponential of matrix
  - * stability of solutions
  - * linearisation of non-linear system near solution / phase plane analysis
- Some orthogonal systems of functions and related differential equations (Bessel, Legendre ...) / Sturm-Liouville
- electrial / mechanical examples, BVP

Convergence (5 classes)

- review of convergence of sequences / series of numbers from INF1
- convergence issues for improper integrals (brief) [Γ-fn.]
- integral test for series $\sum f(n)$
- generalised metric spaces ($\mathbb{R}^n$, function space examples)
- convergence and completeness for metric spaces
- sequences / series of functions
  - * pointwise, uniform, in the mean / L^2 convergence
  - * relations between different forms of convergence, examples
  - * continuity of uniform limit of cts functions
  - * integration of series

- examples / $\frac{d}{dx}\left(\int h(x,y)\,dy\right)$ (definite / indefinite / proper / improper)

Power series (2 classes)

- radius of convergence
- differentiation / integration of power series
- multiplication of power series
- connection to Taylor series
- use for series solutions of some odes

Laplace transforms (2 classes)

- def, examples
- convolution
- H, δ
- applications to d.e.s

Fourier series (3 classes)

- summary of projections from linear algebra
- Bessel inequality
- differentiation/integration of Fourier series
- convergence (pointwise, L^2, uniform)
- Parseval equality
- Fourier transform as limiting notion from Fourier series
- preservation of L^2-norm and inverse Fourier transform

Applications (2 classes) (according to time)

- Schrödinger equ - for H
- wave equation for rectangular boundary
- heat equation 1-D
- wave equation on disc
- ? Green's functions

(Total = 26 classes)