Introduction to Mathematical Computation

Assignment #9

Exercise 9.1 Show that the divided differences are linear maps on functions. That is, prove the equation

$$(\alpha f + \beta g)[x_0, x_1, \ldots, x_n] = \alpha f[x_0, x_1, \ldots, x_n] + \beta g[x_0, x_1, \ldots, x_n].$$

Exercise 9.2 The divided difference $f[x_0, x_1]$ is analogous to a first derivative. Does it have a property analogous to $(fg)' = f'g + fg'$?

Exercise 9.3 Prove the Leibnitz formula:

$$(fg)[x_0, x_1, \ldots, x_n] = \sum_{k=0}^{n} f[x_0, x_1, \ldots, x_k] g[x_k, x_{k+1}, \ldots, x_n].$$

Exercise 9.4 Compare the efficiency of the divided difference algorithm to the original procedure we learned in class for computing the coefficients of a Newton interpolating polynomial.

Exercise 9.5 Find Newton's interpolating polynomial for the following data:

x	1	3/2	0	2
$f(x)$	3	13/4	3	5/3

Use divided differences to calculate the coefficients.

Exercise 9.6 Find an explicit form of the Hermite interpolating polynomial for $k = 2$ (two interpolation points) and $m_1 = m_2 = m$ ($p^{(k)}(x_i) = f^{(k)}(x_i)$ for $k = 0, 1, 2, \ldots, m - 1$).

Exercise 9.7 Find the Hermite interpolating polynomial in the case $m_1 = m_2 =, \cdots = m_k = 2$.
Hint: try

$$p(x) = \sum_{i=1}^{k} h_i(x) f(x_i) + \sum_{i=1}^{k} g_i(x) f'(x_i)$$

with h_i and g_i polynomial of degrees up to $2k - 1$ which satisfy:

$$h_i(x_j) = \delta_{i,j} \qquad g_i(x_j) = 0$$
$$h_i'(x_j) = 0 \qquad g_i'(x_j) = \delta_{i,j}.$$