Introduction to Mathematical Computation

Assignment #8

Exercise 8.1 Let $A = U\Sigma V^T$ be an SVD for the m-by-n matrix A. What are the SVDs for the following matrices:

① $(A^TA)^{-1}$.
② $(A^TA)^{-1}A^T$.
③ $A(A^TA)^{-1}$.
④ $A(A^TA)^{-1}A^T$.

Exercise 8.2 Write an algorithm that gets two vectors, $(x_0, x_1, \ldots, x_n)$ and $(y_0, y_1, \ldots, y_n)$, and a number x, and returns $p(x)$, where p is the interpolating polynomials through the $n+1$ points (x_i, y_i).

Exercise 8.3 Apply Lagrange's interpolation formula to the set of equally spaced pairs:

x	h	$2h$	$3h$
y	y_0	y_1	y_2

to obtain an approximation for $y(x)$ at $x = 0$.

Exercise 8.4 Let $L_i(x)$ be the Lagrange polynomials for the set of point $x_0, \ldots, x_n$, and let $C_i = L_i(0)$. Show that

$$\sum_{i=0}^{n} C_i x_i^j = \begin{cases} 1 & j = 0 \\ 0 & j = 1, \ldots, n \\ (-1)^n x_0 x_1 \cdots x_n & j = n+1, \end{cases}$$

and that

$$\sum_{i=0}^{n} L_i(x) = 1.$$

Exercise 8.5 Suppose that $p(x)$ is the interpolation polynomial of the data:

x	3	7	1	2
y	10	146	2	1

Find a simple expression, in terms of $p(x)$, for the interpolation polynomial of the data:

x	3	7	1	2
y	12	146	2	1