Introduction to Mathematical Computation

Assignment #6

Exercise 6.1 Show that every matrix of the form

$$\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix}$$

$a, b, \neq 0$, has an LU decomposition. Show that even if the diagonal elements of L are 1 the decomposition is not unique.

Exercise 6.2 Show that if $A = LU$ is symmetric then the columns of L are proportional to the rows of U.

Exercise 6.3 Show that every symmetric positive-definite matrix has an LU-decomposition.

Exercise 6.4 Suppose you want to solve the equation $AX = B$, where A is n-by-n and X, B are n-by-m. One algorithm would factorize $A = PLU$ and then solve the system column after column using forward and backward substitution. The other algorithm would compute A^{-1} using Gaussian elimination and then perform matrix multiplication to get $X = A^{-1}B$. Count the number of operations in each algorithm and determine which is more efficient.

Exercise 6.5 Determine the LU factorization of the matrix

$$\begin{pmatrix} 6 & 10 & 0 \\ 12 & 26 & 4 \\ 0 & 9 & 12 \end{pmatrix}.$$

Exercise 6.6 (Computer exercise) Construct in Matlab an n-by-n matrix A (its entries are not important, but make sure it is non-singular), and verify how long its takes to perform the operation `B=inv(A);`. Repeat the procedure for $n = 10, 100, 1000, 2000$.