Introduction to Mathematical Computation

Assignment #1

1. Find the first two terms in the Taylor expansion of $x^{1/5}$ about the point $x = 32$. Approximate the fifth root of 31.999999 using these two terms in the series. How accurate is your answer?

2. The *error function* defined by

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

gives the probability that a trial value will lie within x units of the mean, assuming that the trials have a standard normal distribution. This integral cannot be evaluated in terms of elementary functions.

① Integrate Taylor's series for e^{-t^2} about $t = 0$ to show that

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)\, k!}$$

(more precisely, use the Taylor expansion for e^{-x}).

② Use this series to approximate $\operatorname{erf}(1)$ to within 10^{-7}.

3. Let n be fixed. Show that

$$\sum_{k=0}^{n} x^k = \frac{1}{1-x} + o(x^n)$$

as $x \to 0$.

4. Prove that if $x_n = O(\alpha_n)$ then $\alpha_n^{-1} = O(x_n^{-1})$. Prove that the same holds for the o-relation.