NOTES

PROBLEMS

It has been suggested that this journal might serve as a medium for publishing problems in the area of Logistics – with the idea that other persons might be interested in those problems and might submit comments. Readers are invited to submit brief statements on applied and theoretical problems in Logistics. Address letters to Managing Editor, Naval Research Logistics Quarterly, Office of Naval Research, Washington 25, D. C.

24 December 1959

Dear Sir:

The letters of McShane and Solomon (NRLQ December 1958) and of Davis (NRLQ June 1959) commenting on several articles dealing with the Naval Electronics Allocation Problem, including Kruskal's and my article [1]: "The Coefficients in an Allocation Problem" (NRLQ June 1958), have recently come to my attention. Most of the points raised in these letters were covered in Kruskal's and my article [2]: "Assigning Quantitative Values to Qualitative Factors in the Naval Electronics Problem" (NRLQ March 1959). Still, because of the importance of the questions raised, I think it is worthwhile to go over some of these points in the less technical context of a letter (especially as Davis's letter appeared after [2]).

(1) [1] is not an essay on the Electronics Allocation Problem, but a description of a general approach to subjective allocation problems, i.e., to allocation problems in which the values of the individual assignments are not given numerically at the outset, but must be deduced from "qualitative" information (such as preferences; cf. (9) below). We referred to a greatly simplified version of the Electronics Allocation Problem in [1], to illustrate the general approach without getting lost in a sea of complications. The application to the real-life Electronics Allocation Problem was described in [2], not in [1]. This accounts for the fact that the terminology in [1] was, according to McShane and Solomon, "less clearly defined than it might be," and that "there still remains the need for a sharper definition of the actual problem." I hope that this sharper definition has been provided in [2].

(2) McShane and Solomon state that the MIP "is the basic source of information in the problem." It is true that the MIP is a basic source of information in the problem, and that it is the only basic source readily available in organized form, but the whole complexity of the problem lies in the fact that there are two other basic factors that must be taken into account, and that interact with each other and with the MIP. These factors (goodness of model available for installation and of model already installed) are not as readily available in organized form as the

(5) We used the word "utility," in [1], in the following sense: A utility on a set X of outcomes is a real valued function v on X, with the property that for all outcomes x and y, x is preferred to y implies that $v(x)>v(y)$, and x and y are indifferent implies that $v(x) = v(y)$ (probability combinations are irrelevant in this context). Under this definition, I think it justified in a business problem to call dollar profit a "natural" or "physical" utility. If this definition is rejected, the word "objective function" may be substituted for "utility."

(6) McShane and Solomon make the point that some allocation problems do not possess the complicating features with which we deal. This is all to the good. If an equipment is totally unsuitable for all requirements and ship classes but one and is the only equipment available, and if all positions of this requirement and ship class have voids currently installed, then or course there is no problem; one simply installs the equipments on the ships until one runs out of them. There are, however, cases in which things are not so simple; and it is to these cases that our work has been directed.

(7) Intransitivity would yield inconsistent numerical results, and thus would be sufficient cause to consider our technique inapplicable ([1], pp. 120-121). We did not run across any intransitivities in the experimental phase of our work. I think the intuitive meaning of "preference" precludes intransitivities.

(8) The problem raised by Davis, that the second radar installed on a ship should not have the same value as the first, was discussed under the heading "Multiple Allowances" on p. 12 of [2]. The conclusion we reached there may be expressed in Davis's notation by saying that $(2)a_{11}$ means "the installation of two radar sets on two distinct submarines of the same priority class," and <u>not</u> "the installation of two radar sets on a single ship." Thus (in Davis's words) "we might hope to rule out diminishing marginal utilities." Davis's conclu ion that his plan e could never be preferred to his plan d is not justified, because he has faile take into account factors other than the MIP.

(9) I conclude with a rough explanation of the "logical underpinnings" of our technique. Like Davis's, our approach uses only orderings. Let us concentrate our attention on a fixed string of requirements (a "string" is a set of related requirements that for the purposes of allocation are considered together; cf. [2], p. 3). Basically, we know only that there exists a partial preference order on the set X of all allocation plans for the given string; all this says is that it is sometimes meaningful to say that one plan is preferred to another. Our task is to find a maximal feasible allocation plan, i.e., a feasible plan such that no other feasible plan is preferred to it. (If there is a <u>maximum</u> feasible plan, i.e., a feasible plan preferred or indifferent to all other feasible plans, then every maximal plan will also be a maximum plan; but unless the given preference order is total, a maximum plan may not exist.)

We now make certain assumptions about the preference order; the significant ones are transitivity and an additivity assumption, which says approximately that if plan A is preferred to plan B, and if we add the same assignment a to both A and B, then A+a is still preferred to B+a. (This additivity assumption is closely related to the idea of constant marginal utility.) It

MIP, but that does not mean that we can ignore them (cf. pp. 2-3 of [2]). This fact was recognized when, during the experimental phase of this work, an officer was appointed to decide "token" questions in which the various factors indicated conflicting answers. If the MIP were the only basic source of information, these questions could have been decided trivially.

(3) Neither of the two interpretations of the MIP suggested by McShane and Solomon is the one used by us. Interpretation (a), that "all priority 1 requirements be fulfilled before priority 2", no matter what equipments are installed and no matter how suitable the available model is for the two positions in question, would be too drastic an oversimplification. Interpretation (b), that total "effectiveness" should be maximized, is too vague to be useful. Effectiveness is a derived concept, constructed with the aid of the MIP and other information, and cannot be used to give the intent of the MIP. The interpretation used by us is precisely given on p. 5 of [2]. It says that <u>all priority 1 positions should be filled before any priority 2 positions provided all other factors are equal.</u>

It would be difficult to deny that the MIP says at least this. That it does not say more can be seen from the fact that duly constituted Naval authority has in the past decided "token" problems against the MIP, on the basis of the other factors. Such a procedure can also be easily justified on a common-sense basis: the MIP contains hundreds (if not thousands) of items, all in rank order; it seems intuitively clear that there are some pairs of items that are close to each other on the MIP, that "differ very little" in importance. Now suppose that b_1 and b_2 are positions corresponding to MIP items of this kind, b_1 being preferred to b_2. Let us say that there is a fairly good equipment installed in b_1 that is doing the required job, maybe not in the best possible way, but fairly well; whereas in b_2 there is installed a very poor equipment or no equipment at all. Suppose moreover that the only equipment currently available for installation is eminently suitable for installation in b_2, but comparatively unsuitable for installation in b_1, say because of weight, size, or similar considerations. I think it is fairly clear that the available equipment should be installed in b_2 in spite of the MIP. Now the questions arise, how "near" do b_1 and b_2 have to be in order for this reasoning to be valid? Conversely, how "great" does the difference between the already installed equipments (and/or between the suitabilities of the available equipment to the two positions) have to be to justify going against the MIP? This is the crux of the Naval Electronics Allocation Problem. Simply to say that we will follow the MIP regardless of the other factors is to beg the principal question.

Of course in a given practical allocation problem, all other factors are rarely equal. Nevertheless the MIP, when interpreted as above, is one of several powerful tools that together can be used to arrive at practical allocation plans. I don't think the MIP was ever envisaged as a document that would immediately and single-handedly solve allocation problems.

(4) Questions asked of the "Board" (the individual or group authorized to arrive at allocation decisions) involve realistic situations only ([1], p. 122). Nobody would consider using a VHF transmitter to fulfill the requirement of a surface search radar; questions involving such a possibility would not occur.

can be proved that there then exists a utility v on X (cf. (5) above), with the "linear" property that $v(A+B) = v(A)+v(B)$. Maximization of v over the set of all feasible plans yields a feasible plan that is maximal in the given preference order. Thus if we can succeed in ascertaining v, we have reduced our problem to a standard linear programming problem.

The trouble is that the preference order exists only in the mind of the "Board," and even there it exists only in disconnected pieces. To ascertain the preference order explicitly is out of the question. Even ascertaining the utility v precisely would usually involve asking the board thousands of questions about the preference order, and would be impossible from the practical viewpoint. On the other hand, it is possible to determine v approximately; this is accomplished by using the information contained in the MIP and in the answers to a limited number of "token" questions. If w is an approximation to v, then maximizing w will not necessarily yield a maximal plan, but it will yield a plan that is in a certain sense "close to maximal;" without going into details, we may say roughly that the better the approximation of w to v, the closer the resulting plan will be to maximality. In any case, the plan obtained by maximizing w will be "best possible" with the given information.

It is important to realize that neither the statement of the problem nor its final solution involve the concepts of utility or "military worth." These are derived concepts, used as computational tools in arriving at allocation plans that are maximal or near maximal. Certainly they can have no direct real life interpretation such as "range" or "search radius."

I would like to take this opportunity to thank McShane, Solomon, and Davis for their interesting and stimulating comments on this important problem.

Sincerely yours,

/sgd/ ROBERT J. AUMANN

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