# MATH 200 - SEC 201 - 2010W

## Assignment no. 3

### Due: 9am, Mar 9, 2011

1. Let $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$. Show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2}\,.$$

2. Show that the function

$$u(x, y, t) = \frac{1}{t} e^{-\frac{x^2+y^2}{4t}} .$$

satisfies the *heat equation*:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t} .$$

3. Let

$$F(x, y) = x \arctan(x^2 - y)\,.$$

(a) Assume $g$ is differentiable such that $F(x, g(x)) = 0$ and $g(1) = 1$. Find $g'(1)$.

(b) Find a unit vector $u$, such that directional derivative of $F$ at the point $(1, 1)$ in the direction $u$ is 0.

4. Let $F(x, y, z) = x^2 + y^2 + z^2$.

   (a) Write the gradient $\nabla F$.

   (b) Write the equation of a plane passing through $(x_0, y_0, z_0)$ and orthogonal to $\nabla F(x_0, y_0, z_0)$.

   (c) Find $(x_0, y_0, z_0)$ which belong to the level surface $F(x_0, y_0, z_0) = 1$ for which the plane from (b) passes through the points $(1, 1, 1)$ and $(1, -2, 4)$.