MATH 200 - SEC 104 - 2012W1

Midterm no. 2

1-1:50pm, Oct 12, 2012

Name (Last, First):_____________Oki_____________

Student I.D. Number:__________________________

Signature: __________________________

INSTRUCTIONS: This is a closed-book exam. You may not use any books, notes, papers, calculators, or other aids. Do all work on the sheets provided. There is an extra sheet on the back for scratch work. If you need an extra sheet, raise your hand and one will be provided. If you need more space for your solution, use the back of the sheets and leave an arrow for the grader. Please draw a box around your final answer.

There are 3 questions, each worth 10 points. Explain all your answers. Good Luck!

1. Let $f(x, y) = x^2 + y^2$.

(a) Write the equation of the tangent plane at $(1, 1, 2)$. Simplify your answer.

(b) Find all the points (a, b) for which the tangent plane at $(a, b, f(a, b))$ contains the points $(1, 0, 0)$ and $(0, 2, 0)$.

(a) $f_x = 2x \qquad f_y = 2y$

$f_x(1,1) = 2 \qquad f_y(1,1) = 2 \qquad f(1,1) = 2$

Tangent plane is $z - 2 = 2(x-1) + 2(y-1)$

$$\Updownarrow$$

$$z - 2x - 2y + 2 = 0$$

(b) $f_x(a,b) = 2a \quad f_y(a,b) = 2b \quad f(a,b) = a^2 + b^2$

Tangent plane is

$z - a^2 - b^2 = 2a(x-a) + 2b(y-b)$

has to pass through $(1,0,0)$ and $(0,2,0)$ so

$$\begin{cases} 0 - a^2 - b^2 = 2a(1-a) + 2b(0-b) \\ 0 - a^2 - b^2 = 2a(0-a) + 2b(2-b) \end{cases}$$

equivalently

$$\begin{cases} a^2 + b^2 = 2a \\ a^2 + b^2 = 4b \end{cases}$$

so

$2a = 4b \Rightarrow a = 2b$

this means that

$4b^2 + b^2 = 4b \Rightarrow 5b^2 = 4b \Rightarrow b = \frac{4}{5}$ or $b = 0$

$\Downarrow \qquad \Downarrow$

$a = \frac{8}{5} \qquad a = 0$

$\boxed{\text{answer } \left(\frac{8}{5}, \frac{4}{5}\right) \text{ and } (0,0)}$

2. Let A be such that the function

$$f(x,t) = \frac{e^{-\frac{Ax^2}{t}}}{\sqrt{t}}$$

satisfies the *heat equation* $f_{xx} = f_t$. Find A.

$$f_x = \frac{e^{-\frac{Ax^2}{t}}}{\sqrt{t}}\left(-\frac{2Ax}{t}\right) = -2Ax\, e^{-\frac{Ax^2}{t}}\, t^{-\frac{3}{2}}$$

$$f_{xx} = -2A e^{-\frac{Ax^2}{t}}\, t^{-\frac{3}{2}} - 2Ax\, e^{-\frac{Ax^2}{t}}\, t^{-\frac{3}{2}}\left(-\frac{2Ax}{t}\right)$$

$$= -2A e^{-\frac{Ax^2}{t}}\, t^{-\frac{3}{2}} + 4A^2x^2 e^{-\frac{Ax^2}{t}}\, t^{-\frac{5}{2}}$$

$$f_t = e^{-\frac{Ax^2}{t}}\, t^{-\frac{1}{2}}\left(Ax^2t^{-2}\right) + e^{-\frac{Ax^2}{t}}\left(-\frac{1}{2}t^{-\frac{3}{2}}\right)$$

$$= -\frac{1}{2} e^{-\frac{Ax^2}{t}}\, t^{-\frac{3}{2}} + Ax^2 e^{-\frac{Ax^2}{t}}\, t^{-\frac{5}{2}}$$

In order for $f_{xx} = f_t$ we need that

$-2A = -\frac{1}{2}$ and $4A^2 = A$

Both equations are satisfied by $A = \frac{1}{4}$, and only by it.

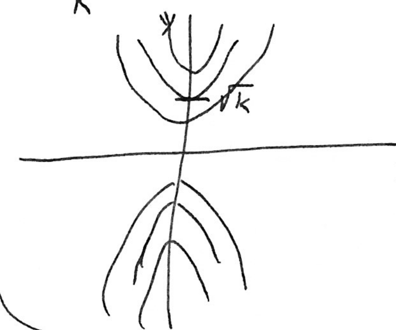
3. Match the functions with pictures of their graphs. Explain your answers.

(a) $f(x, y) = \frac{y^2}{1+x^2}$

(b) $g(x, y) = x^2 \cos(3y)$

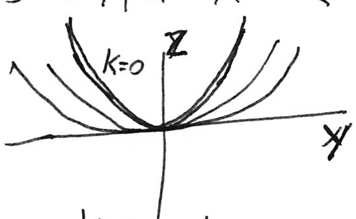
(a) Level curves $\frac{y^2}{1+x^2} = k \Rightarrow \frac{y^2}{k} - x^2 = 1$

hyperbolas



cross sections with $x = k$

$z = \frac{y^2}{1+k^2}$

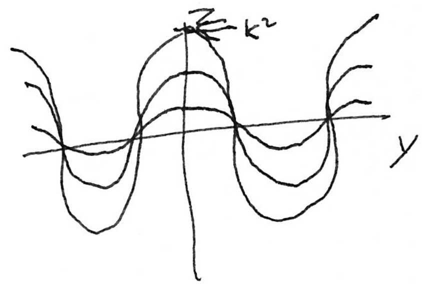


smiling parabolas getting flatter as $|k|$ increases

we see that it fits (H).

(b) cross sections with $x = k$ yield $z = k^2 \cos(3y)$

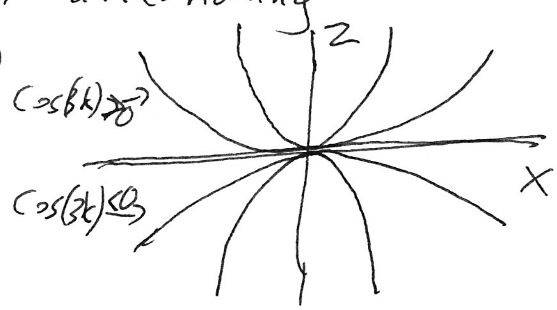
waves with amplitude increasing with $|k|$.



cross sections with $y = k$

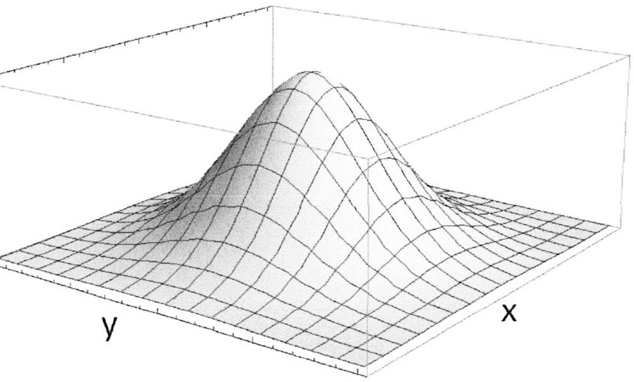
$z = x^2 \cos(3k)$

parabolas with direction alternating according to $\cos(3k)$

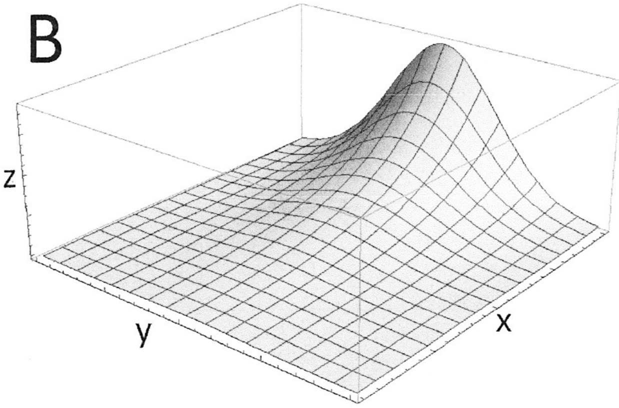


this fits (D)

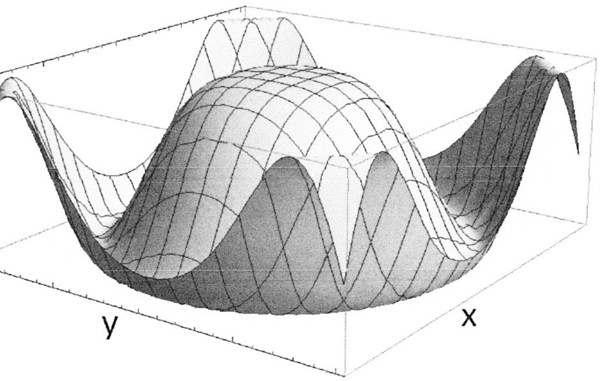
A



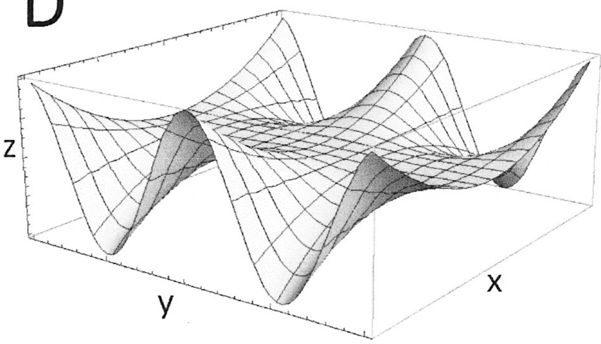
B



C



D



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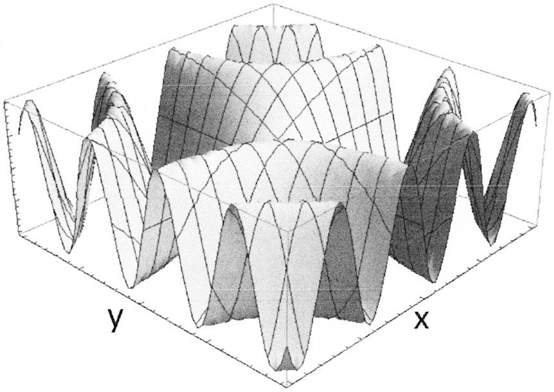
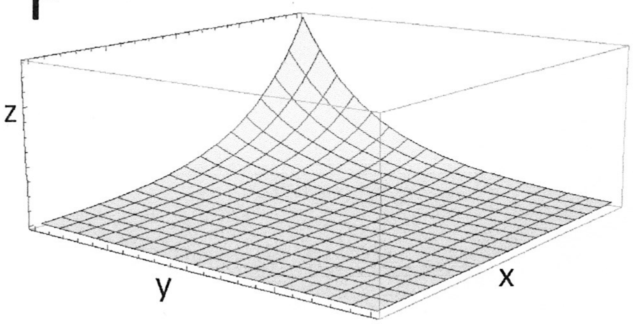
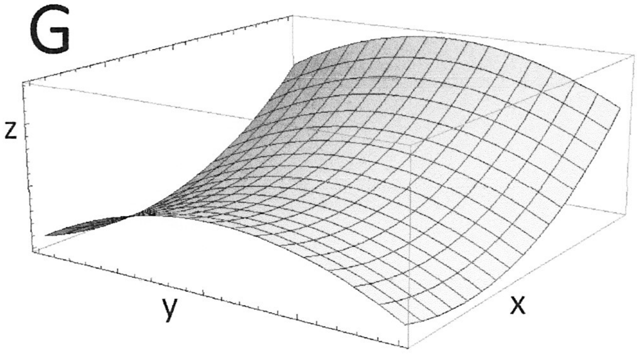


F



G



H