MATH 200 - SEC 104 - 2012W1

Midterm no. 1

1-1:50pm, Sep 24, 2012

Name (Last, First): Ori

Student I.D. Number: ________________

Signature: ________________

INSTRUCTIONS: This is a closed-book exam. You may not use any books, notes, papers, calculators, or other aids. Do all work on the sheets provided. There is an extra sheet on the back for scratch work. If you need an extra sheet, raise your hand and one will be provided. If you need more space for your solution, use the back of the sheets and leave an arrow for the grader. Please draw a box around your final answer.

There are 4 questions, each worth 10 points. Explain all your answers. Good Luck!

1. Let $A = (1,1,1)$, $B = (2,2,3)$ and $C = (1,2,0)$. In the triangle ABC, which angle is the biggest?

$\vec{AB} = \langle 2,2,3\rangle - \langle 1,1,1\rangle = \langle 1,1,2\rangle$

$\vec{AC} = \langle 1,2,0\rangle - \langle 1,1,1\rangle = \langle 0,1,-1\rangle$

$\vec{AB}\cdot\vec{AC} = \langle 1,1,2\rangle \cdot \langle 0,1,-1\rangle = 0+1-2 = -1 < 0$

hence, the angle $\measuredangle BAC$ is obtuse and must be the biggest – no need to check the other angles.

2. Let u, v be two nonzero vectors, such that $\text{comp}_u v = \text{comp}_v u$. What can you deduce about the vectors u and v?

$$\text{comp}_u v = \frac{u \cdot v}{|u|}$$

$$\text{comp}_v u = \frac{u \cdot v}{|v|}$$

$$\frac{u \cdot v}{|u|} = \frac{u \cdot v}{|v|}$$

$$\Downarrow$$

$$|v| \, u \cdot v = |u| \, u \cdot v$$

$$\Downarrow$$

$$u \cdot v \, (|v| - |u|) = 0$$

$$\Downarrow$$

Either $u \cdot v = 0$, that is u and v are othogonals

or $|u| = |v|$, that is u and v have the same size.

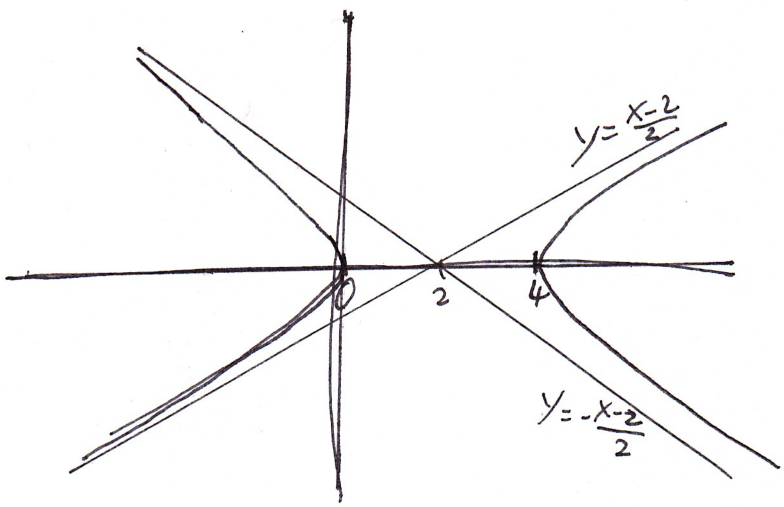
3. Draw the graph of the equation $x^2 = 4y^2 + 4x$.

The equation can be re-written as

$$\frac{(x-2)^2}{2^2} - y^2 = 1$$

Hence, it is a hyperbola, centered around $(2,0)$ with $a=2$ $b=1$, and it does not intersect the line $x=2$, since $-y^2=1$ has no solutions.

Its asymptotes are $y = \pm \frac{x-2}{2}$



4. Let P be the plane defined by $x + 2y - 2z + 5 = 0$ and let A be the point $(3, 2, 3)$.

(a) What is the distance between A and P?

(b) Let L be the line going through A and orthogonal to the plane P. Find a vector equation for L.

(c) Let B be the point where L intersects P. Find B.

(d) What is the length of the line segment AB?

(e) Explain the relation between the answers to part (a) and (d).

(a) $$\frac{|3+2\cdot 2-2\cdot 3+5|}{\sqrt{1^2+2^2+(-2)^2}} = \frac{6}{3} = 2$$

(b) A line orthogonal to P has the same direction as a normal to P, which is $\langle 1,2,-2\rangle$

$L: \; u = \langle 3,2,3\rangle + t\langle 1,2,-2\rangle$

(c) The parametric equations for the line are

$x = 3+t \quad y = 2+2t \quad z = 3-2t$.

Substituting in the equation for P yields

$(3+t)+2(2+2t)-2(3-2t)+5=0$

$6+9t=0$

$t=-\frac{2}{3}$

so $B=(3-\frac{2}{3}, 2-\frac{4}{3}, 3+\frac{4}{3}) = (2\frac{1}{3}, \frac{2}{3}, 4\frac{1}{3})$

(d) The distance is $\sqrt{(3-2\frac{1}{3})^2+(2-\frac{2}{3})^2+(4\frac{1}{3}-3)^2} =$

$= \sqrt{(\frac{2}{3})^2+(\frac{4}{3})^2+(\frac{4}{3})^2} = 2$

(e) B is the closest point to A on P since it is where the perpendicular line L intersects P. Hence the distance AB is equal to the distance AP.