MATH 200/253 - SEC 922 - 2012S1

Quiz no. 1

4:00-4:25pm, May 11, 2012

Name (Last, First):___________Ori___________________

Student I.D. Number:_______________________________________

Signature: ____

INSTRUCTIONS: This is a closed-book exam. You may not use any books, notes, papers, calculators, or other aids. Do all work on the sheets provided. There is an extra sheet on the back for scratch work. If you need an extra sheet, raise your hand and one will be provided. If you need more space for your solution, use the back of the sheets and leave an arrow for the grader. Please draw a box around your final answer.

There are 4 questions, each worth 10 points. Explain all your answers. Good Luck!

1. Let $u = 5\vec{i} + 2\vec{j} - \vec{k}$ and $v = 7\vec{i} - 2\vec{j} - \vec{k}$. Find a unit vector in the direction of $u - 2v$.

$u = \langle 5, 2, -1 \rangle$

$V = \langle 7, -2, -1 \rangle$

$u - 2v = \langle -9, 6, 1 \rangle$

$|u-2v| = \sqrt{81 + 36 + 1} = \sqrt{118}$

$\widehat{(u-2v)} = \left\langle -\frac{9}{\sqrt{118}}, \frac{6}{\sqrt{118}}, \frac{1}{\sqrt{118}} \right\rangle$

2. Consider the set of all points in the plane $P = (x, y)$ such that the distance from P to $A = (-1, 1)$ is twice the distance from P to $B = (2, -5)$. Show that this set is a circle and find its center and radius.

$$\sqrt{(x-(-1))^2+(y-1)^2} = 2\sqrt{(x-2)^2+(y-(-5))^2}$$

$$(x+1)^2+(y-1)^2 = 4(x-2)^2+4(y+5)^2$$

$$x^2+2x+1+y^2-2y+1 = 4x^2-16x+16+4y^2+40y+100$$

$$3x^2-18x+3y^2+42y+114=0$$

$$x^2-6x+y^2+14y+\cancel{3}38=0$$

$$x^2-6x+9+y^2+14y+49=20$$

$$(x-3)^2+(y+7)^2=20$$

$$\sqrt{(x-3)^2+(y+7)^2}=\sqrt{20}$$

This is the circle with radius $\sqrt{20}$, centred at $(3,-7)$

3. Let L be the line described by the parametric equations

$$x = t \quad y = 1 + 2t \quad z = 3 + 3t$$

and let P be the plane defined by $3x - 3y + z + 4 = 0$.

(a) Show that L and P are parallel.

(b) Find the distance between L and P.

vector equation for L: $u = \underbrace{\langle 0,1,3\rangle}_{u_0} + t\underbrace{\langle 1,2,3\rangle}_{v}$

normal to P is $n = \langle 3,-3,1\rangle$

(a) $n \cdot v = 3 - 6 + 3 = 0$

n & v are orthogonal so L is parallel to P

(b) we can take the distance of u_0 from P.

$$D = \frac{|3 \cdot 0 - 3 \cdot 1 + 1 \cdot 3 + 4|}{\sqrt{9+9+1}} = \frac{4}{\sqrt{19}}$$

4. Let u, v be two nonzero vectors, such that $\text{comp}_u v = \text{comp}_v u$. Is it necessarily true that $|u| = |v|$? Prove or give a counterexample.

$$\text{comp}_u v = \frac{u \cdot v}{|u|}$$

$$\text{comp}_v u = \frac{u \cdot v}{|v|}$$

$$\frac{u \cdot v}{|u|} = \frac{u \cdot v}{|v|}$$

$$\Downarrow$$

either $|u| = |v|$ or $u \cdot v = 0$

so 2 orthogonal vectors of different lengths would be a counterexample.

For example: $u = \langle 1, 1 \rangle$ $v = \langle 2, -2 \rangle$.

$|u| = \sqrt{2}$ $|v| = 2\sqrt{2}$

$\text{comp}_u v = \text{comp}_v u = 0$

Scratch sheet