MATH 253 - SEC 104 - W2011T1

Quiz no. 3

2:00-2:20pm, Nov 1, 2011

Name (Last, First): Ovi

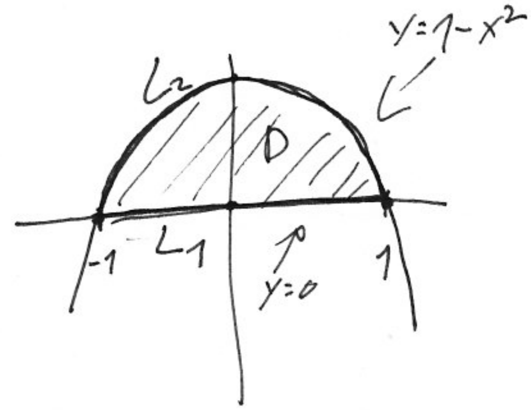
Student I.D. Number:

Signature:

INSTRUCTIONS: This is a closed-book exam. You may not use any books, notes, papers, calculators, or other aids. Do all work on the sheets provided. There is an extra sheet on the back for scratch work. If you need an extra sheet, raise your hand and one will be provided. If you need more space for your solution, use the back of the sheets and leave an arrow for the grader. Please draw a box around your final answer.

There are 3 questions, each worth 10 points. Explain all your answers. Good Luck!

1. Let $F(x,y) = xy$ and let $D = \{(x,y) \mid 0 \le y \le 1 - x^2\}$. Find the absolute minimum and maximum values of F in D.



$F_x = y = 0$

$F_y = x = 0$

$(0,0)$ is a CP.

$L_1 = \{(x,y) : y = 0 \quad -1 \le x \le 1\}$

$L_2 = \{(x,y) : y = 1-x^2 \quad -1 \le x \le 1\}$

$g(x) = F(x,0) \quad -1 \le x \le 1$

$= 0$

all the point $(x,0)$ are candidates.

$g(x) = F(x, 1-x^2) \quad -1 \le x \le 1$

$= x - x^3$

$g'(x) = 1 - 3x^2 = 0$

$x = \pm \frac{1}{\sqrt{3}}$

$y = \frac{2}{3}$

$\left(\frac{1}{\sqrt{3}}, \frac{2}{3}\right) \quad \left(-\frac{1}{\sqrt{3}}, \frac{2}{3}\right)$

$F(x,0) = 0 \qquad F\left(\frac{1}{\sqrt{3}}, \frac{2}{3}\right) = \frac{2}{3\sqrt{3}} \qquad F\left(-\frac{1}{\sqrt{3}}, \frac{2}{3}\right) = -\frac{2}{3\sqrt{3}}$

max

min

2. Let $f(x, y, z)$ be differentiable at $(1, 1, 1)$ and such that $\nabla f(1, 1, 1) = \langle 1, -2, -1\rangle$. Let $g(t) = f(e^t, 1/(1+t), \cos(t))$. Find $g'(0)$.

By the chain rule

$$g'(t) = F_x h_1'(t) + F_y h_2'(t) + F_z h_3'(t)$$

$$F_x(h_1(t), h_2(t), h_3(t))$$

want $t=0$

$$F_x(h_1(0), h_2(0), h_3(0))$$

$$F_x(1,1,1) = 1$$

$$F_y(1,1,1) = -2$$

$$F_z(1,1,1) = -1$$

$h_1(t) = e^t$.

$h_2(t) = \frac{1}{1+t}$

$h_3(t) = \cos t$

at $t=0$

$h_1'(t) = e^t \quad = 1$

$h_2'(t) = -\frac{1}{(1+t)^2} \quad = -1$

$h_3'(t) = -\sin(t) \quad = 0$

$$g'(0) = 1\cdot 1 + (-2)(-1) + (-1)0 = 3$$

3. Suppose that the equation $F(x,y,z)=0$ implicitly defines each of the three variables x,y and z in term of the other two :$z=f(x,y)$, $y=g(x,z)$ and $x=h(y,z)$. Assume that F is differentiable at (x_0,y_0,z_0) and that $F_x(x_0,y_0,z_0)$, $F_y(x_0,y_0,z_0)$ and $F_z(x_0,y_0,z_0)$ are all nonzero. Show that

$$f_x(x_0,y_0)\,g_z(x_0,z_0)\,h_y(y_0,z_0)=-1\ .$$

B_y

$$f_x(x_0,y_0) = -\frac{F_x(x_0,y_0,z_0)}{F_z(x_0,y_0,z_0)}$$

$$g_z(x_0,z_0) = -\frac{F_z(x_0,y_0,z_0)}{F_y(x_0,y_0,z_0)}$$

$$h_y(y_0,z_0) = -\frac{F_y(x_0,y_0,z_0)}{F_x(x_0,y_0,z_0)}$$

$$\left(-\frac{\cancel{F}_x}{\cancel{F}_z}\right)\cdot\left(-\frac{\cancel{F}_z}{\cancel{F}_y}\right)\left(-\frac{\cancel{F}_y}{\cancel{F}_x}\right)=-1$$