MATH 253 - SEC 104 - W2011T1

Quiz no. 2

2:00-2:20pm, Oct 4, 2011

Name (Last, First):_____Ovi_____

Student I.D. Number:_____

Signature: _____

INSTRUCTIONS: This is a closed-book exam. You may not use any books, notes, papers, calculators, or other aids. Do all work on the sheets provided. There is an extra sheet on the back for scratch work. If you need an extra sheet, raise your hand and one will be provided. If you need more space for your solution, use the back of the sheets and leave an arrow for the grader. Please draw a box around your final answer.

There are 3 questions, each worth 10 points. Explain all your answers. Good Luck!

1. Let $u = \langle 2, 3, 0 \rangle$ and $v = \langle 3, -1, 7 \rangle$.

   (a) Find a *unit* vector orthogonal to both u and v.

   (b) Is the solution to (a) unique? Explain.

(a) $w = u \times v = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 3 & -1 & 7 \end{vmatrix} = \langle 21, -14, -11 \rangle$

$|w| = \sqrt{21^2 + (-14)^2 + (-11)^2} = \sqrt{441 + 196 + 121} = \sqrt{758}$

$\hat{w} = \frac{w}{|w|} = \frac{1}{\sqrt{758}} \langle 21, -14, -11 \rangle$

(b) no. $-\vec{w}$ is also a solution to (a).

2. Let L be the line defined by $\langle 0,1,3\rangle + t\langle 1,2,3\rangle$ and let P be the plane defined by $3x - 3y + z + 4 = 0$.

   (a) Show that L and P are parallel.

   (b) Find the distance between L and P.

(a) L and P are parallel if and only if the direction of L is orthogonal to the normal of P.

$\langle 1,2,3\rangle \cdot \langle 3,-3,1\rangle = 3 - 6 + 3 = 0$

(b) $\langle 0,1,3\rangle$ is on L

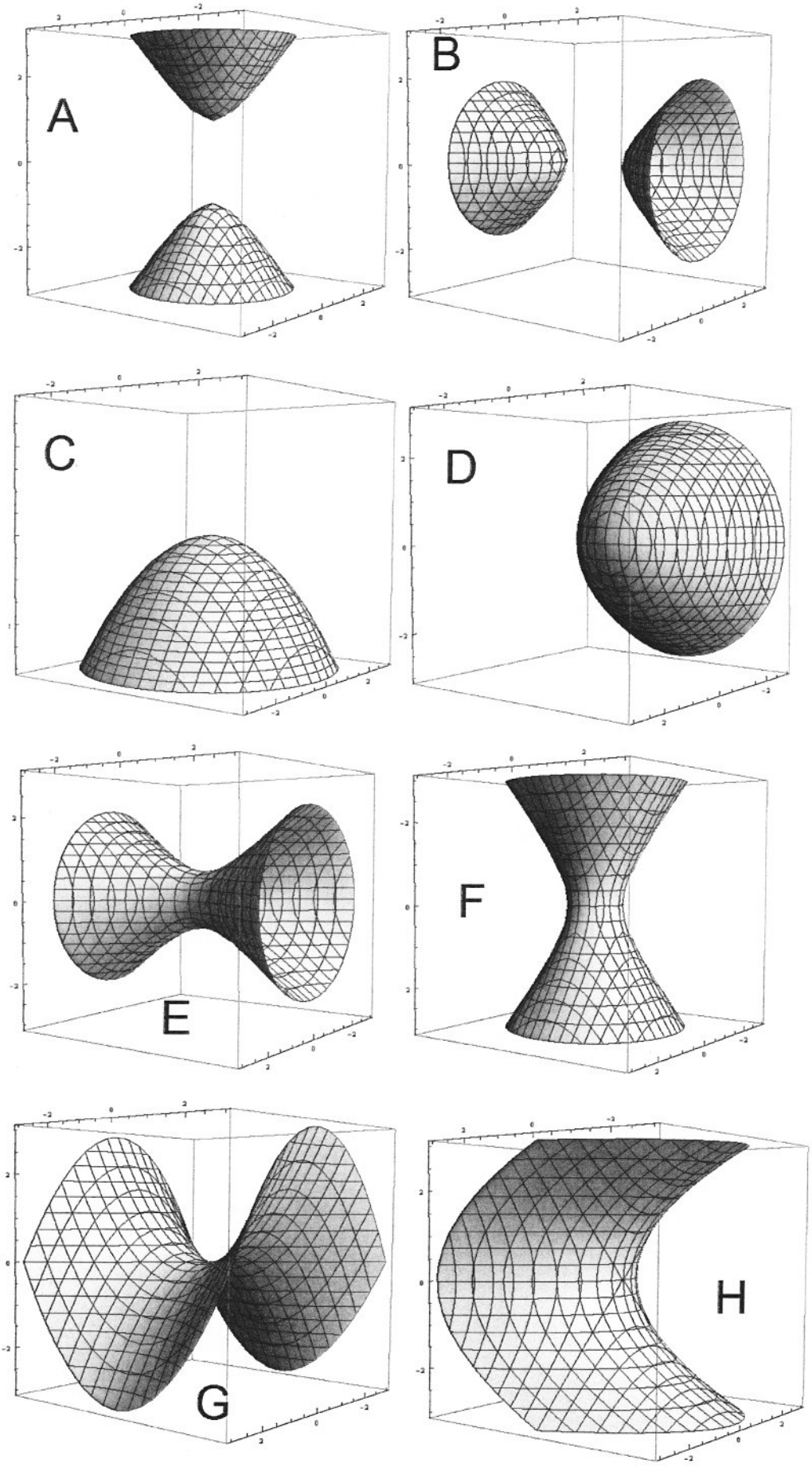
$$\frac{|3\cdot 0 - 3\cdot 1 + 1\cdot 3 + 4|}{\sqrt{3^2 + (-3)^2 + 1^2}} = \frac{4}{\sqrt{19}}$$

3. Consider the equation $2x^2 - y^2 + 2z^2 + 1 = 0$. Which of the following is a picture of the surface defined by this equation? Explain.

If we look at intersection with the plane $y = k$ we get

$2x^2 + 2z^2 = k^2 - 1$.

This is a circle of radius $\sqrt{\frac{k^2-1}{2}}$ when $|k| \geq 1$ and empty set when $|k| < 1$ so we get a hyperboloid of 2 sheets around the y-axis: B.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