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A NON-COOPERATIVE AXIOMATIZATION OF THE CORE

ABSTRACT. We treat a class of multi-person bargaining mechanisms based on games in coalitional form. For this class of games we identify properties of non-cooperative solution concepts, which are necessary and sufficient for the equilibrium outcomes to coincide with the core of the underlying coalitional form game. We view this result as a non-cooperative axiomatization of the core. In contrast to most of the literature on multi-person bargaining we avoid a precise specification of the rules of the game. Alternatively, we impose properties of such games, which give rise to a large class of mechanisms, all of which are relevant for our axiomatization.

KEY WORDS: Multilateral bargaining, Core, Non-cooperative axiomatization, Game equilibrium

1. INTRODUCTION

The purpose of this paper is to provide a characterization of the core of coalitional form games as a non-cooperative solution concept on a class of bargaining games. Our objective is to apply multilateral bargaining without precisely specifying the rules of a single (extensive form) bargaining game, but rather by providing a list of properties of such games, which will induce a class of bargaining procedures. In particular, unlike almost all the literature on bargaining theory, we intend to avoid the terminology of proposals and responses. We feel that real-life bargaining situations often involve actions that have very little to do with proposals or responses. A player may come up with declarations and threats, send payoff-irrelevant messages (cheap talk), or even take actions to disrupt the communication channels between two of his opponents. Obviously, such behavior cannot be allowed when players' actions consist of proposals and responses only. This approach is also meant to deal with the criticism that bargaining theory is often accused of. Namely, the fact

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that the results obtained are typically non-robust with respect to the specifications of the rules of games.

The basic properties of the bargaining procedures in our domain is that they involve a sequence of ‘agreement attempts’. Each agreement attempt is initiated by a player and each potential participant can cause the current agreement attempt to break down.

The characterization of the core in our class of games will be based on four properties of the solution concept:

1. Subgame Perfection,
2. Subgame Consistency,
3. Payoff-Oriented Choice Rule, and
4. Maximality.

Subgame perfection simply requires that the solution concept of the corresponding extensive form game should be a subgame perfect equilibrium. The requirement of subgame consistency imposes a stationarity property on the players’ behavior. Specifically, if a player is facing two subgames that are equivalent from a strategic point of view, then he should act in the same manner in these two subgames. The condition of payoff-oriented choice rule imposes another simplification: roughly, each player has a reservation price, which he considers at any decision node where he is able to determine the bargaining outcome; at such a decision node he will choose to cause a breakdown of the agreement attempt if and only if all possible agreements yield him less than his reservation price. The condition requires that the reservation price of a player may depend only on the set of players active in current negotiations. Finally, the maximality condition simply means that we consider the set of *all* strategy combinations satisfying the three axioms stated above.

The main result of the paper is that these four properties yield a non-cooperative characterization for the core of the underlying coalitional form games for any bargaining model within our class.

Our approach extends several other approaches in the literature on the relation between non-cooperative bargaining and the core. Chatterjee et al. (1993) analyzed the efficiency of non-cooperative bargaining outcomes and their relation to the core. Perry and Reny (1994) established an implementation for the core using a continuous time model and a feature in the bargaining mechanism that allows players to renegotiate agreements. Okada (1992) characterized the core using properties similar to ours in a proposal model which is one of specific bargaining procedures in our class. Moldovanu and Winter (1994) show that a simple proposal model implements the core on the class of convex games. Winter (1996) uses the property of mechanism robustness of the equilibrium outcome to establish the implementation of the core. Finally, Winter (1997) discusses core implementation in the framework of committees' decision making where players have ordinal preferences over an abstract set of alternatives. In contrast to this literature, we provide here a non-cooperative axiomatization of the core rather than an implementation. We identify a set of properties for the non-cooperative solution concept which is necessary and sufficient for core implementation in our class of bargaining mechanisms. Obviously, this set of axioms is not the only possible axiomatization of the core, as is also the case with many cooperative axiomatizations of various cooperative solution concepts. In particular, Peleg (1986) established an axiomatization of the core using the reduced game property, which is a purely cooperative approach. Our axiomatization does, however, focus on the question of how simple does players' equilibrium behavior need to be in order to obtain core implementation for every bargaining mechanism within our class.

2. THE RESOURCES

We set $N = \{1, 2, \cdots, n\} (n \geq 2)$ to denote the set of players (bargainers). Any subset (including N) of N is called a coalition of players. As in most of the literature on multilateral bargaining, we treat n-person games in coalitional form as the primitive of the model. Hence, the underlying opportunities of the individuals are given by a function v associating a real number $v(S)$ with every coalition $S \subseteq N$. The number $v(S)$ stands for the total resources the coalition S can allocate among its members. Formally, an n-person game in coalitional form with transferable utility (called a TU-game) is defined by a pair (N, v), and v is called the characteristic function of a TU-game.[1] For the empty coalition ϕ, we set $v(\phi)$ to be zero for our convenience. In what follows, the characteristic function v itself is also called a TU-game if no confusion arises.

A payoff vector for N is an n-dimensional real vector, $x = (x_i : i \in N)$, and we write $x(S) = \sum_{i \in S} x_i$ for any coalition S of N.[2] These notations are applied to a payoff vector for a coalition T whenever no confusion arises. The restricted game of a TU-game (N, v) on S is a TU-game (S, v^S) where the player set is S and the characteristic function v^S is the restriction of v on S.

We impose the following standard assumptions on the domain of all TU-games.

(1) v is zero-normalized, i.e., $v(\{i\}) = 0$ for $i = 1, 2, \cdots, n$
(2) v is super-additive, i.e., $v(S \cup T) \geq v(S) + v(T)$ for any disjoint subsets S, T of N.

DEFINITION 1.

(1) A payoff vector $x = (x_i : i \in S)$ for a coalition S is called *feasible (with respect to S)* if $x(S) \leq v(S)$. The set of all feasible payoff vectors for S in a TU-game (N, v) is denoted by $X(S, v)$.
(2) The *core* of (N, v) is defined by

$$C(N, v) = \{x \in X(N, v) | x(S) \geq v(S) \quad for\ all \quad S \subseteq N\}.$$

(3) A TU-game (N, v) is *totally balanced* if every restricted game (S, v^S) in it has a non-empty core $C(S, v^S)$.
(4) A payoff vector $x = (x_i : i \in N)$ for N is called *semi-stable* if it satisfies
    (i) $x(S) \geq v(S)$ for all $S \subseteq N$, and
    (ii) for every i in N, there exists some coalition $S \ni i$ such that $x(S) = v(S)$.

The notions of feasible payoff vectors, core and totally balancedness are standard in cooperative game theory. The notion of a semi-stable payoff vector was introduced by Albers (1975) and investigated by Selten (1981) and Bennett (1983). This notion is based on the following idea: The payoff x_i of every player i is interpreted as his demand or his aspiration level in coalitional bargaining. Condition (i) means that no player can increase his demand so that some coalition will satisfy the demands of its members. In this sense, the payoff (demand) vector x is maximal. Condition (ii) means that every player can find some coalition S, which includes himself, such that meeting the demands of all members exactly divides total utility $v(S)$.

It is already apparent from the definition of semi-stable payoff vectors that this notion is related to that of the core. This relation is made formal in the following result.

PROPOSITION 1. *For a payoff vector* $x = (x_i : i \in N)$ *for* N, *the following conditions are equivalent.*

(1) x *is in the core* $C(N, v)$.

(2) x *is feasible with respect to* N, *and*

$$x_i = \max_{S: i \in S \subseteq N} \{v(S) - x(S - \{i\})\} \quad for\ all \quad i \in N. \quad (1)$$

(3) x *is semi-stable, and* $x(N) = v(N)$.

Proof. Note that

$$x_i \geq \max_{S: i \in S \subseteq N} \{v(S) - x(S - \{i\})\} \quad for\ all \quad i \in N$$

if and only if $x(S) \geq v(S)$ for all $S \subseteq N$. Then the proof is straightforward. □

Proposition 1 exposes an important property of the core (and of the semi-stable payoffs). (1) implies that every payoff vector $x = (x_i : i \in N)$ in the core is attained by a certain type of non-cooperative payoff-maximizing behavior on the part of each player. That is, given that all other players receive their payoffs in x, every player attempts to maximize his dividend $v(S) - x(S - \{i\})$ in a coalition S which he joins.

The value on the right-hand side of (1) is called the *maximum payoff to player* i *at* x, and is denoted by $m_i(x)$. We can easily see that every semi-stable payoff vector is characterized by a fixed point of the mapping $m(x) = (m_1(x), \cdots, m_n(x))$ from R^n_+ to itself where R^n_+ is the non-negative orthant of the n-dimensional Euclidean space R^n.

We now turn to the domain of non-cooperative bargaining games based on an underlying coalitional form game. As mentioned in the introduction, our aim in this paper is to avoid full specification of the rules of a single game. We will impose a list of properties, shared by a class of extensive form games, that will determine the domain of our bargaining games.

3. PROPERTIES OF THE NON-COOPERATIVE BARGAINING GAMES

The issues negotiated in all bargaining games include both coalition formation and the distribution of payoffs. In each bargaining game negotiations take place over (possibly) infinitely many sessions. Each session consists of a finite number of 'agreement attempts'. If an agreement is reached in some session within some coalition, this coalition leaves the game with the agreed payoff distribution and the game continues with the remaining players. In this section, we will determine the class of our bargaining games by describing the structure of these games. We will start with the notion of 'agreement attempts', which are the elementary building blocks of the games. We will then move to describe sessions, which consists of a finite sequence of agreement attempts, and finally describe the structure of the whole game which consists of an infinite sequence of sessions.

Let $N^t (\subseteq N)$ be the set of players active in session t $(= 1, 2, \cdots)$ where $N^1 = N$, and consider any session t for which N^t is non-empty. We define by $A(N^t) = \{(S, x^S) | N^t \supseteq S \neq \phi,\ x(S) \leq v(S)\}$ the set of all possible agreements in session t.

An *agreement attempt* G is a finite-length extensive game form with perfect information.[3] In what follows, we use standard terminology such as nodes, edges (actions) and paths in a game tree. Each terminal node of the agreement attempt G is associated with an element of $A(N^t) \bigcup \{F\}$ where F stands for failure to reach an agreement in that attempt. When a terminal node associated with an agreement (S, x^S) is reached, it means that the agreement (S, x^S) is reached in G. Such a terminal node is called an agreement node. A terminal node with symbol F is simply called F-node. We say that a player selects F-node if he chooses an action leading directly to this node, and we interpret it as the failure of the current agreement attempt.

We now impose four properties of agreement attempts to determine the domain of non-cooperative bargaining games. Let i be the first player to act in an agreement attempt G and let o_i be i's first decision node. We will say that player i is an initiator in G. An agreement attempt with initiator i is denoted by G_i. We assume that the initiator can "terminate" the current session, in which case the next session $t + 1$, identical to session t, starts.

(A1) (Agreement) For any $(S, x^S) \in A(N^t)$ with $i \in S$, there exists a path p from o_i to an agreement node (S, x^S).

This property means that every feasible agreement can be reached (with the help of other players) if the initiator is a member of its coalition.

(A2) (The ability to interrupt) All decision nodes on path q from o_i to every agreement node (T, y^T) are associated with players in T only. Furthermore, every player in T has a decision node on path q leading directly to an F-node.

This property means that the set, say Q, of all players acting on a path from the initial node o_i to every agreement node (T, y^T) is exactly equal to coalition T. Since the initiator i acts at o_i, the property implies that the initiator is a member of T. We, however, remark that the result in the paper holds true in a more general case that either $Q = T$ or $Q = T \cup \{i\}$ is allowed. The last part of (A2) imposes that every member of coalition T can interrupt the agreement (T, y^T).

(A3) (Paths to agreement) Let r be a path from a decision node d_j for player j to an agreement node (S, x^S) with $j \in S$ and let a_j be player j's action at d_j on path r. Then, for any agreement node (T, y^T) following d_j via a_j, it must be true that $S = T$ and $x_j^S = y_j^T$.

This requirement imposes that player i by initiating the agreement (S, x^S) guarantees the payoff x_i and the formation of S at any agreement that is reached within the current attempt.

(A4) (The last player) For any path p from o_i to an agreement node (S, x^S) with $S \supset \{i\}$, there exists a unique player $j\,(\neq i)$ with a decision node on p at which any action leads immediately

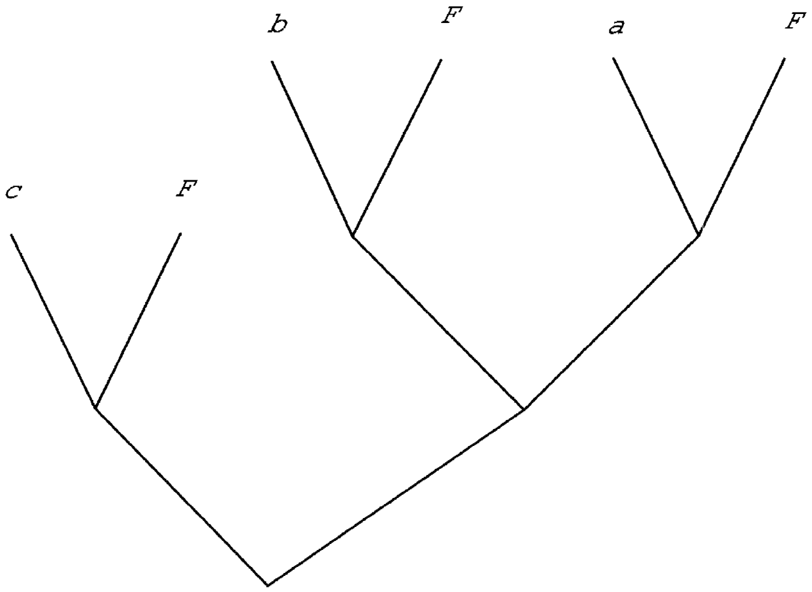

Figure 1.

to a terminal node; moreover, one course of action leads to F-node. We call such a player the *last player* on path p.

This requirement says that any agreement involving at least two players including the initiator i is reached by a final step in which some player (other than the initiator) either makes an action that leads to such agreement or causes the agreement to fail. Figure 1 shows a game tree satisfying (A4). In the figure, symbols a, b and c represent possible agreements and F represents failure of agreement. (A4) rules out the game tree in Figure 2. If the left edge is chosen at decision node x, then another decision node y, not a terminal node, is reached. The player acting at the decision node x is a last player but this player has an action (leading to y) which neither determines an agreement nor causes its failure.

We proceed the formulation of our bargaining game by describing the nature of a session in the game. A *session* is a game tree composed of a finite number of agreement attempts. At the beginning of a session an initiator, i, is chosen by a predetermined rule and an agreement attempt G_i is played. If an agreement attempt G_i fails due to an action by some player j, then the session continues by moving to a new agreement attempt G_j with initiator j. More

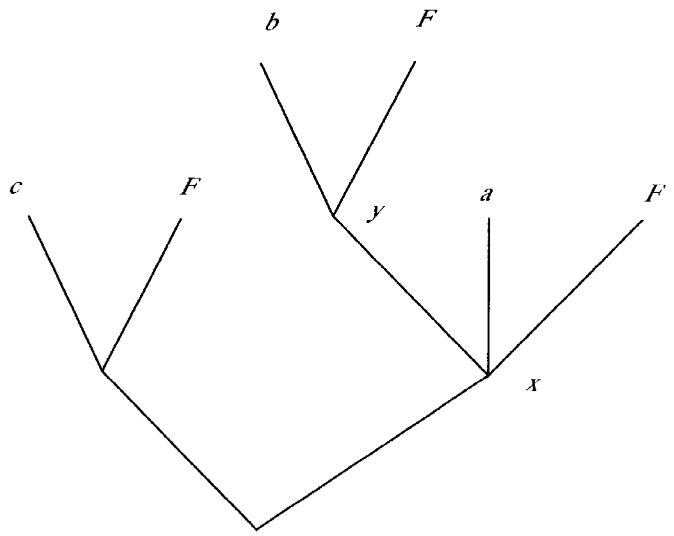

Figure 2.

specifically:

(i) The initiator in the first agreement attempt is chosen exogenously from N^t independently of the history of previous sessions (one can also use a probability distribution[4] on N^t to make this choice).
(ii) Each session consists of $K (\geq 2)$ agreement attempts unless agreement is reached earlier, in which case the session terminates upon reaching an agreement.

Our result of core implementation is not sensitive to the choice of an initiator in every session. This is due to the fact that altering the initiator gives rise to a new bargaining game which is still within the same class of models on which our axiomatization applies.

To complete our description of the domain of the bargaining games we describe the structure of the whole game.

A *bargaining game* is composed of a sequence of (a potentially infinite number of) sessions in the following way. If session $t (= 1, 2, \cdots)$ is terminated with no agreement, then session $t + 1$ is identical to session t. If an agreement (S, x^S) is reached in session t, then session $t + 1$ is some session with the set of active players $N^{t+1} = N^t - S$ where $N^1 = N$. The game terminates when an agreement (S, x^S) is reached in session t where $N^t = S$. Each player i receives the payoff agreed upon. If the game does not terminate (there is an infinite sequence of sessions), each remaining player receives zero-payoff, and the rest receive the payoffs agreed upon. We assume that every player has perfect information on the game play whenever he makes a choice.

For a TU-game v, let $B(v)$ be the class of all extensive form games that satisfy (A1)–(A4), (i) and (ii) with respect to v. We now give examples of two concrete bargaining games within our domain.

(1) Proposal model (see Okada, 1992). The initiator proposes a pair (S, x^S) of a coalition involving himself and a payoff vector for its members, and all the other players in S either accept or reject it sequentially. If all the players accept it, (S, x^S) is agreed, and players not in S continue negotiations. If one of the players rejects it, he can make a counter-proposal and the same process is repeated. If no agreement is reached after a predetermined number of proposals, negotiations start all over again.

(2) Demand-commitment model. This game is a version of Selten (1992). The initiator announces a pair (S, x_i^S) where S is a coalition containing i and x_i^S is i's payoff demand. Then, all other players j in S also demand their payoffs x_j^S sequentially. If the demand vector $x^S = (x_j^S)$ is feasible with respect to S, then (S, x^S) is agreed upon. All players not in S continue negotiations. If the demand vector x^S exceeds $v(S)$, then the first player k for which this happens (player k caused the failure of an agreement) comes up with a new 'demand' (T, x_k^T). This goes on for a fixed finite number of demands. If no agreement is reached within this horizon, bargaining begins once again from the start.[5]

Every bargaining game in our class is composed of sequence of sessions, each consisting of a finite number of agreement attempts. Intuitively, one should observe the following scenario when trying to interpret this structure. The negotiating parties assemble daily for a certain period of time to discuss an agreement. This daily meeting constitutes a single session in which a fixed number of agreement attempts are undertaken. If no agreement is reached, the parties convene a day later to resume negotiations in a new session. This new session is held under precisely the same rules as the previous one.

Because the set of actions in our class of bargaining games are abstract, they can include actions that are not necessarily offers and responses. In the proposal model and the demand commitment model, we can add either cheap-talk messages that are not payoff relevant or some actions that have payoff implications. For example, they may include actions that constitute a commitment not to accept certain offers, either when a seller reveals his outside option while negotiating the price, or when a politician makes a public announcement that states the limit of his willingness to compromise in some international negotiations.

4. A NON-COOPERATIVE AXIOMATIZATION OF THE CORE

We present an axiomatization of the core for an n-person coalitional game from the point of view of non-cooperative game theory. In what follows, the (initial) player set $N^1 = \{1, \cdots, n\}$ is fixed. Let V be the set of all TU-games on N that are zero-normalized, superadditive and totally balanced. Let $B = \bigcup_{v \in V} B(v)$. That is, B is the class of all bargaining games in extensive form derived from TU-games v in V.

Let $\Gamma \in B$. Since Γ is represented by an extensive game with perfect information, we can define a pure strategy combination $\sigma = (\sigma_1, \cdots, \sigma_n)$ for players in Γ. We restrict our treatment to pure strategies, which (as it turns out) suffices for the purpose of axiomatizing the core. For a pure strategy combination $\sigma = (\sigma_1, \cdots, \sigma_n)$ for players in Γ, the (expected) payoff vector for the players is denoted by $H(\sigma) = (H_1(\sigma), \cdots, H_n(\sigma))$.[6]

DEFINITION 2. A *solution* φ of the n-person coalitional bargaining problem is a function that assigns to every bargaining game $\Gamma \in B$ a set $\varphi(\Gamma)$ of pure strategy combinations $\sigma = (\sigma_1, \cdots, \sigma_n)$ for Γ.

In order to present the axioms of the solution φ, we need to introduce two definitions. Let Γ and Γ' be two extensive games with perfect information, and let K and K' be the set of all nodes in the game trees of Γ and Γ', respectively. An *isomorphism* from Γ and Γ' is a one-to-one mapping g from K onto K' satisfying the following three properties.

(I1) g preserves the 'tree' structure of K. That is, for any two nodes x and y in K, node x precedes node y if and only if $g(x)$ precedes $g(y)$ in K'.

Note that by (I1), the mapping g from K to K' induces a one-to-one mapping f from the set E of all edges in Γ to the set E' of all edges in Γ', i.e., if an edge e connects x and y in Γ, then the corresponding edge $f(e)$ connects $g(x)$ and $g(y)$ in Γ'. For every pure strategy combination $\sigma = (\sigma_1, \cdots, \sigma_n)$ in Γ, the one-to-one mapping f from E to E' naturally induces a pure strategy combination $\sigma' = (\sigma'_1, \cdots, \sigma'_n)$ in Γ'. We call σ' the pure strategy combination in Γ' induced from σ by the isomorphism g from Γ to Γ', and we denote σ' by $g(\sigma)$.

(I2) A node x is player i's decision node in Γ if and only if $g(x)$ is player i's decision node in Γ'. The same is true for chance nodes in Γ and Γ' and every two corresponding edges at chance nodes in Γ and Γ' are assigned the same probabilities.

(I3) Let H and H' denote the expected payoff functions of Γ and Γ', respectively. For every pure strategy combination $\sigma = (\sigma_1, \cdots, \sigma_n)$ in Γ, $H(\sigma) = H'(g(\sigma))$.

Two extensive games Γ and Γ' are called *isomorphic* if there exists an isomorphism g from Γ to Γ'. Note that for any Γ in B, any two subgames in Γ which start with the beginning of a session and have the same set of active players are isomorphic.

Let $\Gamma \in B$ be a bargaining game and let e be an action at a decision node for some player i in a session of Γ. Assume that there exists a path following the edge e to an agreement node (S, x^S) with $i \in S$. Then, by property (A3), we can associate the edge e with a unique real value $u(e)$ defined by x_i^S. The value $u(e)$ can be interpreted as the payoff that player i can obtain if an agreement is made by taking the action e. If the edge e has no path to an agreement node, then we set $u(e) = -\infty$ for convenience.

We are now ready to state the axioms needed for our characterization of a solution φ. We use $\sigma^* = (\sigma_1^*, \cdots, \sigma_n^*)$ in the axioms to denote a pure strategy combination in $\varphi(\Gamma)$. Whereas our axioms are formally imposed on a solution φ of the n-person coalitional bargaining problem, a function assigning to every bargaining game $\Gamma \in B$ a set $\varphi(\Gamma)$ of pure strategy combinations for Γ, they provide a refinement of a Nash equilibrium point for each bargaining game in the class B.

AXIOM 1 (subgame perfection). If $\sigma^* \in \varphi(\Gamma)$, then σ^* is a subgame perfect equilibrium point of Γ.

Subgame perfection is a fundamental rationality requirement for behavior in extensive games.

AXIOM 2 (subgame consistency).[7] If $\sigma^* \in \varphi(\Gamma)$, then σ^* satisfies the following property: Let Γ' and Γ'' be any two isomorphic subgames of Γ and let $\sigma^*(\Gamma')$ and $\sigma^*(\Gamma'')$ be pure strategy combinations on Γ' and Γ'' induced by σ^*, respectively. Then $\sigma^*(\Gamma'') = g[\sigma^*(\Gamma')]$ for some isomorphism g from Γ' to Γ''.

Subgame consistency requires that players behave identically in every two isomorphic subgames of the extensive game. This requirement generalizes the stationarity requirement for equilibria used in an almost every multilateral bargaining model (see Perry/Reny, 1994; Chatterjee et al., 1993; Okada, 1992; and Winter, 1996).

AXIOM 3 (payoff-oriented choice rule). If $\sigma^* \in \varphi(\Gamma)$, then σ^* satisfies the following property. Each player i has a list $(p_i^S)_{i \in S \subseteq N}$ of 'reservation prices' such that if player i is the last player[8] on a path in some agreement attempt with a set S of active players, then player i makes the following choice: Let A_i be the set of all agreement nodes reached immediately by player i's action at his last decision node, and let $u_i^* = \max\{u_i(a) | a \in A_i\}$.
If $p_i^S \leq u_i^*$, then player i takes some action yielding payoff u_i^*.
If $p_i^S > u_i^*$, then player i takes the action leading to F-node (i.e., player i breaks down the current agreement attempt).

This axiom restricts the behavior of the last player i in every agreement attempt with every set S of active players as follows. If there exists an action for player i that yields an agreement whereby he is paid more than a certain threshold level p_i^S, then among these actions player i will take the one that yields him the highest payoff; otherwise, he will break the agreement attempt. The threshold level p_i^S may be interpreted as a 'reversation price' or a 'minimum acceptance level' of player i in specific contexts of negotiations. Generally, such a reservation price of the player may depend on a whole history of bargaining. The axiom requires that the reservation price of every player depends only on the set of active players in negotiations, and thus that the same choice rule should be applied in all agreement attempts and in all sessions, as long as the set of active players are unchanged. Note that Axiom 3 is applied only to the last players in all agreement attempts. As we will show in the proof of the main theorem, it is sufficient for us to restrict only the behavior of the last player.

To understand the role of Axiom 3 better, it is helpful to consider its implication on the proposal model (Okada 1992) given in the last section. Let $x^* = (x_1^*, .., x_n^*)$ denote the expected payoff vector for players in a subgame perfect equilibrium point satisfying Axiom 2 (subgame consistency). Let us consider the response rule of the last player i in agreement attempts when all players are active in negotiations. By backward induction and the subgame consistency, we can see that the responder i accepts any proposal in the last agreement attempt if and only if he is offered a payoff higher than or equal to his equilibrium payoff x_i^* (more precisely, he is indifferent between accepting and rejecting if the offer is equal to x_i^*.) Since the same logic can be applied to all other responders, a proposer j in the last agreement attempt can obtain optimally his maximum payoff $m_j(x^*)$ at the equilibrium payoff vector x^*. Again, backward induction implies that the last responder i in the second-to-last agreement attempt accepts any proposal if and only if he is offered not less than his maximum payoff $m_i(x^*)$. Hence, Axiom 3 requires that $x_i^* = m_i(x^*)$ for all i in N. Thus, a subgame perfect equilibrium point satisfying Axioms 2 and 3 prescribes the simple behavior of the last responder that he accepts any proposal in every agreement attempt if and only if he is offered not less than his equilibrium payoff. Finally, since for every player there exists some path on which he becomes the last player in an agreement attempt, we can restrict all players' reservation prices effectively by Axiom 3.

Several remarks on Axiom 3 should be made here. First, both Axiom 2 and Axiom 3 impose a limit on the complexity (history-dependency) of players' strategies, without violating the full rationality of players that is incorporated in Axiom 1 (subgame perfection). Axiom 3 is not, however, implied by Axiom 2. For example, consider the two situations that a player acts as the last player in the first agreement attempt and in the last agreement attempt in some session. The two subgames starting at these decision nodes are not isomorphic even if the set of active players is identical, because these two subgames have different numbers of remaining agreement attempts. Therefore, Axiom 2 does not impose any restriction on the behavior of the player at these two decision nodes, but Axiom 3 imposes the same choice rule at these nodes. In view of multiplicity of subgame perfect equilibria satisfying the full rationality (Axiom 1), Axioms 2 and 3 assert that players coordinate on those equilibria which have simple properties, i.e., consistency and payoff orientation.

Secondly, Axiom 3 is related to the notion of Markov perfect equilibrium point in the literature (for example, see Fudenberg and Tirole 1991). Roughly, a Markov perfect equilibrium point is a subgame perfect equilibrium point consisting of Markov strategies, strategies which may depend only on 'payoff-relevant' state variables. Since the set of active players determines the characteristic function of the (sub)game in our bargaining game, it can be regarded as a payoff-relevant state variable in our game.

Finally, we emphasize that the collection of reservation prices p_i^S is not exogenous. It is endogenously determined by the strategies σ_i^* that players use. Axiom 3 requires that the strategies that players use in equilibrium give rise to such reservation prices, and indeed different equilibria may correspond to different collections of reservation prices.

AXIOM 4 (maximality). Let Ψ be any solution on B satisfying Axioms 1- 3. Then, $\varphi(\Gamma) \supseteq \Psi(\Gamma)$ for every Γ in B.

Axiom 4 guarantees that *all* pure strategy combinations for Γ satisfying subgame perfection, subgame consistency, and payoff-oriented choice rule are considered.

It is clear that the four axioms are independent, namely, no one axiom is derived from the other axioms. We have already mentioned that Axiom 3 is not implied by Axiom 2. Axiom 2 is not implied by Axiom 3, either, because Axiom 3 restricts only the behavior of the last player in every agreement attempt. Without Axiom 4, a solution φ may select a proper subset of all subgame perfect equilibria satisfying Axioms 2 and 3 in a bargaining game.

In what follows, we will often abuse terminology by attributing axioms to strategy combinations. Thus a pure strategy combination σ is said to satisfy an axiom if σ is a selection from a solution satisfying that axiom.

For $\Gamma \in B$ and a (non-cooperative) solution φ, we denote by $H[\varphi(\Gamma)]$ the set of all expected payoff vectors $H(\sigma)$ sustained by some pure strategy combination σ in $\varphi(\Gamma)$.

We are now ready to state the main result of the paper.

THEOREM . *Let φ be a solution of the n-person coalitional bargaining problem satisfying Axioms 1–4. Then,*

$$H[\varphi(\Gamma)] = C(N, v)$$

for every TU-game $v \in V$ and every bargaining game $\Gamma \in B(v)$.

EXAMPLE (unanimous bargaining). There are n players who negotiate on the division of a cake of size 1. An agreement requires unanimous acceptance by all the players. Formally, the relevant n-person TU-game has $v(N) = 1$ and $v(S) = 0$ for all proper coalitions S of N. The core of the underlying game is simply the set of all Pareto efficient allocations, i.e., $\{x = (x_1, \cdots, x_n) | x_1 + \cdots + x_n = 1, x_i \geq 0 \ (i = 1, \cdots, n)\}$. Consider now a bargaining game in our class B. We will demonstrate why every equilibrium satisfying our axioms must yield a Pareto efficient outcome for such a game. Consider an equilibrium outcome $y = (y_1, \cdots, y_n)$. We will first argue that for each player i, i's reservation price p_i (as defined in Axiom 3) is precisely his equilibrium payoff y_i. Consider the last agreement attempt at some session of the game and suppose that player i is the last player to move in this agreement attempt (see (A4)). Axiom 2 (subgame consistency) implies that he can ensure himself at least y_i by taking an action that will cause the breakdown of the current agreement attempt and that will induce a new session.

This shows by Axiom 1 (subgame perfection) that $p_i \geq y_i$. Now if this inequality is strict for some i, then Axiom 3 (payoff-oriented choice rule) requires that this player should cause the breakdown of every agreement that yields him z_i where $y_i < z_i < p_i$. By doing so, he will obtain payoff y_i. But this violates Axiom 1 (subgame perfection) because he obtains the higher payoff z_i by accepting the agreement. The second step is to show that with such reservation prices the equilibrium outcome must be Pareto efficient. Suppose that the equilibrium outcome y is not Pareto efficient. Indeed Axiom 3 implies that the initiator of an agreement attempt can improve upon the equilibrium outcome y if he takes an action leading to an allocation $w = (w_1, \cdots, w_n)$ that Pareto-dominates y. This is because $y_j < w_j$ for all $j = 1, \cdots, n$ means that if player i initiates such an agreement, then no other player will block it as every player gets more than his reservation price.[9]

In what follows, we will show that the same type of arguments described for the case of unanimous bargaining above hold true in the general setup of coalitional bargaining. A rough argument runs as follows. Suppose that an equilibrium outcome x is not in the core, and thus that $\sum_{i \in S} x_i < v(S)$ for some coalition S. Then, we have $x_i < m_i(x)$. Under the fact that every player j's ($j = 1, \cdots, n$) reservation price p_j^N is equal to x_j, player i can guarantee himself the maximum payoff $m_i(x)$ at x, being the initiator of the last agreement attempt in some session. Considering his optimal action in the second-to-last agreement attempt, we can show from the payoff-oriented choice rule that $m_i(x) \leq p_i^N$. This, together with $x_i < m_i(x)$, implies $x_i < p_i^N$, which is a contradiction.

Formally, we will prove our theorem with the help of four lemmas. Let $\Gamma \in B(v)$. Let $\sigma = (\sigma_1, \cdots, \sigma_n)$ be a pure strategy combination for Γ satisfying Axioms 1 and 2 , and $H(\sigma) = x = (x_1, \cdots, x_n)$ be the expected payoff vector for σ in Γ. Let $H_i(\sigma | o_i)$ be player i's conditional expected payoff for σ at the initiative node o_i in a final agreement attempt in session $t (= 1, 2, \cdots)$ with the player set N (failure to reach an agreement closes the current session). Axiom 2 implies that $H_i(\sigma | o_i)$ is independent of the history leading to this decision node.

LEMMA 1. *Let* $\Gamma \in B(v)$ *and let* φ *satisfy Axioms 1 and 2. Then, for every* $\sigma = (\sigma_1, \cdots, \sigma_n)$ *in* $\varphi(\Gamma)$ *we have*

$$H_i(\sigma | o_i) = m_i(x) \quad \textit{for all} \quad i \in N$$

where $x = H(\sigma)$ *and* $m_i(x) = \max_{S: i \in S \subseteq N} \{v(S) - x(S - \{i\})\}$.

Proof. (1) By way of contradiction, suppose first that $H_i(\sigma|o_i) < m_i(x)$. Since the super-additivity of v implies that $x(N) \leq v(N)$, we note that $x_i \leq m_i(x)$. Then the two cases are possible: $x_i = m_i(x)$ and $x_i < m_i(x)$. Consider the case of $x_i = m_i(x)$. By the rule of an agreement attempt, the initiator i can terminate the current session at o_i. If he does so, the next session starts and the initiator i obtains the payoff x_i (by Axiom 2), which is greater than $H_i(\sigma|o_i)$ by supposition. This contradicts Axiom 1. Next, assume $x_i < m_i(x)$. Let S^* be a solution of the maximization problem

$$\max_{S: i \in S \subseteq N} \{v(S) - x(S - \{i\})\}.$$

If $S^* = \{i\}$, we have $m_i(x) = v(\{i\}) = 0$ since v is zero-normalized. By property (A1), there exists a path from the initial node o_i to the agreement node $(\{i\}, 0)$. By property (A2), no other players have their decision nodes on this path. This means that player i can reach the agreement node $(\{i\}, 0)$ from o_i by his own actions. This fact, together with Axiom 1, implies that $H_i(\sigma|o_i)$ is greater than or equal to 0. This contradicts the supposition of $H_i(\sigma|o_i) < m_i(x) = 0$. Next, consider the case that S^* contains at least two members including player i. Find $\varepsilon > 0$ such that $m_i(x) - \varepsilon > \max\{H_i(\sigma|o_i), x_i\}$. Consider an agreement (S^*, y^{S^*}) such that

$$y_i^{S^*} = m_i(x) - \varepsilon, \quad y_j^{S^*} = x_j + \frac{\varepsilon}{|S^*| - 1}, \quad j \in S^*, \quad j \neq i. \tag{2}$$

By Property (A1), there exists a path p from the initial node o_i to the agreement node (S^*, y^{S^*}). Let $j (\neq i)$ be the last player on path p. If player j takes the action on path p, he obtains the payoff $y_j^{S^*}$, which is greater than x_j from (2). If he selects F-node, Axiom 2 implies that he will obtain payoff x_j in the next session. Therefore, Axiom 1 implies that player j never selects F-node under σ. Let k be the second-to-last player on path p. If player k selects the action on the

path p, he obtains y_k^{S*} by Property (A3) because the last player never selects F-node. Since $y_k^{S*} > x_k$ from (2), player k never selects F-node under σ, either. By repeating the same argument, we can show by Property (A2) that none of the players on path p except the initiator i at o_i selects F-node under σ. Again, Property (A3) implies that the initiator i can obtain y_i^{S*} by selecting the action path p at o_i. Since $y_i^{S*} > H_i(\sigma|o_i)$ from (2) and the selection of ε, this contradicts Axiom 1.
(2) Suppose now that $H_i(\sigma|o_i) > m_i(x)(\geq 0)$. Noting that $x_i \leq m_i(x)$, this supposition with Axiom 2 implies that σ induces a (unique) path q from o_i to some agreement node (S, y^S) with $S \supset \{i\}$ and $y_i^S = H_i(\sigma|o_i)$. Thus,

$$y_i^S > m_i(x) \geq v(S) - x(S - \{i\}). \tag{3}$$

On the other hand, we have

$$v(S) - y^S(S - \{i\}) \geq y_i^S \tag{4}$$

since y^S is a feasible payoff allocation for S. (3) and (4) imply $x(S - i) > y^S(S - \{i\})$. Therefore, there exists some $j(\neq i)$ in S such that $x_j > y_j^S$. Let j be the last player on path q for which this inequality holds. We can assume that player j has an action leading immediately to an F-node. Property (A2) guarantees the existence of such a player j. If player j selects F-node, he obtains x_j (by Axiom 2). If he selects the action on path q, his payoff is y_j^S because the agreement node (S, y^S) is reached under σ. Therefore, Axiom 1 implies that he never selects the action on path q under σ. This contradicts that path q is induced by σ. □

LEMMA 2. *Let* $\Gamma \in B(v)$ *and let* $\sigma = (\sigma_1, \cdots, \sigma_n)$ *be a pure strategy combination in* Γ *satisfying Axioms 1, 2 and 3. Then,* $x_j = p_j^N = m_j(x)$ *for all* $j \in N$ *where* $x = H(\sigma)$ *and* p_j^N *is a reservation price of player* j *in an agreement attempt with the player set* N.

Proof. (1) First, we prove $x_j = p_j^N$ for all $j \in N$ by examining the condition of an optimal choice that player j makes as the last player in the final agreement attempt with the player set N. For simplicity of notation, we write $p_j^N = p_j$. Suppose $x_j > p_j$. Choose any $i \in N$ with $i \neq j$. We can find an agreement $(\{i, j\}, y)$ such that

$$y_i + y_j = v(\{i, j\}) \quad \text{and} \quad p_j < y_j < x_j \tag{5}$$

(note that y_i may be negative here). Consider the final agreement attempt where player i is an initiator. By Properties (A1) and (A4), we can find a path p from o_i to the agreement node $(\{i, j\}, y)$ and player j is the last player on path p. Let z be the last node for player j on p. By Properties (A3), (A4) and (5), we can see that

$$\max\{u(e) | e \text{ is an action for player } j \text{ at } z\} = y_j.$$

Recall that $u(e)$ is the payoff that player j can obtain if an agreement node is reached by taking an action e. Since $p_j < y_j$, Axiom 3 requires that under σ player j should select some action e^* at z with $u(e^*) = y_j$. Then, player j obtains the payoff y_j. On the other hand, if player j selects F-node at z, he obtains a payoff x_j which is higher than y_j from (5). This contradicts Axiom 1.
(2) Suppose now that $x_j < p_j$. Choose any $i \in N$ with $i \neq j$. We can find an agreement $(\{i, j\}, y)$ such that

$$y_i + y_j = v(\{i, j\}) \quad \text{and} \quad x_j < y_j < p_j. \tag{6}$$

By the same argument as (1), we can find a path q from o_i to the agreement node $(\{i, j\}, y)$ and player j is the last player on path q. Let z be the last node for player j on p. By Properties (A3), (A4) and (6), we obtain

$$\max\{u(e) | e \text{ is an action for player } j \text{ at } z\} = y_j.$$

Since $y_j < p_j$, Axiom 3 requires that player j should select F-node at z under σ. By doing so, player j obtains payoff x_j. On the other hand, if he selects some action e^* at z with $u(e^*) = y_j$, then he obtains y_j which is higher than x_j from (6). This contradicts Axiom 1.
(3) Consider the second-to-last agreement attempt. If player j selects F-node as the last player, he can become the initiator in the last agreement attempt by the rule of the bargaining game Γ. Lemma 1 shows that player j's continuation payoff is $m_j(x)$ at his initiative node in the last agreement attempt. Therefore, by repeating the same

arguments as (1) and (2) above, replacing x_j with $m_j(x)$, we can prove $m_j(x) = p_j$. Thus, we can prove $x_j = p_j = m_j(x)$. □

LEMMA 3. *Let* $\Gamma \in B(v)$. *For any pure strategy combination* $\sigma = (\sigma_1, \cdots, \sigma_n)$ *for* Γ *satisfying Axioms 1, 2 and 3, the expected payoff vector* $H(\sigma)$ *is in the core* $C(N, v)$.

Proof. Let $H(\sigma) = (x_1, \cdots, x_n)$. By the rule of the bargaining game Γ and the super-additivity of v, we can see that $x(N) \leq v(N)$. By Property (A2), every player can guarantee at least zero payoff in Γ by selecting F-node. Therefore, it must hold that $x_i \geq 0$ for every i in N. Proposition 1 and Lemma 2 imply now that x is in $C(N, v)$ □

LEMMA 4. *Let* $\Gamma \in B(v)$. *For any payoff vector* $x^N = (x_1^N, \cdots, x_n^N)$ *in the core* $C(N, v)$, *there exists a pure strategy combination* $\sigma^* = (\sigma_1^*, \cdots, \sigma_n^*)$ *for* Γ *satisfying Axioms 1, 2 and 3 with* $H(\sigma^*) = x^N$.

Proof. (1) For every restricted game (M, v^M) of (N, v) with $M \subseteq N$, let $x[M]$ be some core allocation of (M, v^M) and let $x[N] = x^N$. A core-selection $x[M]$ exists since v is totally balanced. Putting $x = x[M]$, let $G(x)$ be an extensive game obtained from an agreement attempt with the active player set M by replacing all F-nodes of the game tree with the core-payoff vector x. Since $G(x)$ is a finite-length extensive game with perfect information, we can apply the backward-induction procedure to $G(x)$. Let τ be a subgame perfect equilibrium point of the finite game $G(x)$ in pure strategies satisfying subgame consistency. Since the game is finite, at least one such equilibrium always exists. For every $j \in M$, set $p_j^M = x_j$ as his 'reservation price' for the active player set M in Axiom 3. Let j be the last player on some path p. Suppose

$$p_j^M \leq \max\{u(e)|e \text{ is an action for player } j\text{'s last node}\}.$$

If this inequality is strict, then any subgame perfect equilibrium prescribes player j to choose an agreement. If there is an equality, then for some subgame perfect equilibrium point player j will choose an agreement. Moreover, if

$$p_j^M > \max\{u(e)|e \text{ is an action for player } j\text{'s last node}\},$$

then player j will choose F-node in every subgame perfect equilibrium point since $p_j^M = x_j$. The above arguments show that there exists a subgame perfect equilibrium point τ for $G(x)$ satisfying Axioms 2 and 3.

(2) By Property (A1), there exists at least one path from the initiative node o_i to an agreement node (M, x). Let p^* be any path of this kind. We will show that the action at every decision node on path p^* is an optimal action under τ. Let us first remark that if a player selects the choice on path p^* at his decision node, he will necessarily obtain payoff x_i by Property (A3) and the construction of $G(x)$. We will check the optimality of the choice on path p^* at every decision node.

Case (1): At the initiative node o_i.

Suppose that an agreement node (S, y^S) with $y_i^S > x_i$ can be realized under τ if player i deviates from path p^*. Let q be the path from o_i to the agreement node (S, y^S). Then, all actions on q except the one at o_i are selected under τ. Since

$$y^S(S - \{i\}) + y_i^S \leq v(S) \leq x(S - \{i\}) + x_i,$$

there exists k in $S,\ k \neq i$, such that $y_k^S < x_k$. Let k be the first player of this kind on path q. Without loss of generality, we can assume from Property (A2) that player k can choose F-node. If player k chooses F-node, he will obtain the payoff x_k by the construction of $G(x)$. On the other hand, if player k selects the action on q, he will obtain the payoff y_k^S by Property (A3) whenever agreement is reached. Indeed, from our supposition, he will obtain y_k^S under τ. Comparing his payoffs for these two choices, we can see that player k will not select the action on path q under τ. This contradicts the fact that q can be realized under τ. Hence, even if player i selects an action at o_i leading to an agreement node (S, y^S) with $y_i^S > x_i$, the agreement node can not be realized under τ and player i' s payoff is at most x_i. Thus, player i' s action on path p^* at o_i is optimal under τ.

Case (2): At a non-initiative node on p^*.

Suppose player j selects an action on path q leading to an agreement node (S, y^S). Then, by Property (A3), we must have $S = M$. Applying the same argument as in case (1), we can show that even

if player j selects an action leading to an agreement node (M, y^M) with $y^M_j > x_j$, agreement (M, y^M) can not be reached under τ. Thus, player j's action on path p^* is optimal under τ.
(3) Let us fix a path p^* of $G(x)$ from o_i to the agreement node (M, x). We construct a pure strategy combination $\tau^* = (\tau^*_i : i \in M)$ for $G(x)$ satisfying

(i) τ^* prescribes to remain on path p^*,
(ii) τ^* coincides with τ at all decision nodes on any path other than p^*.

τ^* may be different from τ at decision nodes on path p^*, but this possible change in players' choices never violates the optimality condition of τ off path p^* because of the tree structure of $G(x)$. Furthermore, this never affects the optimality of path p^* under τ because if player j selects the action on p^* he will obtain the payoff x_j regardless of all the other succeeding players' choices (which may be changed in τ^* from τ).
(4) We now construct the pure strategy combination $\sigma^* = (\sigma^*_1, \cdots, \sigma^*_n)$ for the whole game Γ by imposing that in every session with every player set M, all players in M employ the pure strategy combination τ^* constructed in (3). Using the arguments in (1), (2) and (3), we get that σ^* satisfies $H(\sigma^*) = x$, and that

σ^* prescribes an optimal action at every decision node in Γ.

This property implies that σ^* is a subgame perfect equilibrium point of Γ. Clearly, σ^* satisfies Axioms 2 and 3. □

We can now complete the proof of the theorem.

Proof of theorem. It is clear that a solution φ for the n-person coalitional bargaining problem satisfies Axioms 1-4 if and only if for every bargaining game Γ in the class B, $\varphi(\Gamma)$ consists of all pure strategy combinations that satisfy Axioms 1-3. Thus, the proof of the theorem follows immediately from Lemmas 1 to 4. □

To conclude this section, we provide two remarks about the theorem.

REMARK 1. The proof of the theorem has used the condition of totally balancedness which is a rather standard condition in the literature on core implementation (e.g. Perry and Reny 1994). This condition is only used in the construction of a stationary (subgame consistent) subgame perfect equilibrium point. If the game is not totally balanced, then following the departure of some coalition from the game the resulting (restricted) TU-game may have an empty core, and the corresponding subgame possesses no equilibria satisfying the properties of Axioms 1, 2 and 3. Note also that if the game is not balanced, i.e., when the core is empty, then our result implies that there exists no equilibrium satisfying our axioms. Finally we remark that if we drop any one of our four axioms, then our equivalence result will not hold.

REMARK 2. In the seminal paper on multilateral bargaining, Selten (1981) uses a proposal model which is not within the class B of our extensive form games. Our theorem does not hold in Selten's proposal model. The protocol of a proposal model is roughly the following: At the beginning of the game a player, say i, makes a proposal. A proposal is a pair (S, x^S) where $S \ni i$ is a coalition and x^S is a feasible allocation for S. All the members of S then respond sequentially according to an exogenously specified order. If all players accept the offer, the game terminates and (S, x^S) becomes the agreement. If some member of S rejects the offer, then this player submits a new proposal. Note that this model is based on ONE session with (potentially) infinitely many agreement attempts, while the games in our class consist of sequence of sessions, each with a finite number of agreement attempts. Consequently our main theorem does not apply to Selten's model. Indeed an equilibrium outcome of Selten's model does not necessarily yield a core allocation even when the core is non-empty.

Consider for example a simple one seller -two buyers game, i.e., $N = \{1, 2, 3\}$ and $v(\{1, 2\}) = v(\{1, 3\}) = v(\{1, 2, 3\}) = 1$, $v(\{2, 3\}) = 0$, and $v(\{i\}) = 0 (i = 1, 2, 3)$. The unique point in the core allocates the whole unit to the seller, i.e., (1, 0, 0). However we can sustain the agreement $(\{1, 2\}, (1/2, 1/2))$ as a subgame perfect equilibrium point of Selten's game using the following strategy combination when player 1 is the first proposer:

(1) Player 1 proposes $(\{1, 2\}, (1/2, 1/2))$, accepts any offer of at least 1/2 and reject all other offers.
(2) Player 2 uses the same strategy as player 1.
(3) Player 3 proposes $(\{1, 3\}, (1/2, 1/2))$, accepts any offer of at least 1/2 and rejects all other offers.

A (stationary subgame perfect) equilibrium outcome of Selten's game may not even be Pareto-efficient. Moldovanu and Winter (1995) demonstrate that the core allocations can be characterized as the equilibrium outcomes in such one session model if one either imposes an additional condition of order-independence on the strategy combination, or alternatively one restricts the analysis to a subclass of coalitional form games, e.g., convex games. We remark that models like Selten's (1981) or Moldovanu and Winter's (1995) do not belong to our class of bargaining games as they lack the session structure we have in this paper. Therefore Axiom 3 does not restrict players' behavior effectively to obtain core implementation. One needs other type of restrictions to obtain core implementation in such models like the order independence condition proposed by Moldovanu and Winter (1995).

5. CONCLUSION

We have presented a non-cooperative characterization of the core of coalition form games. In contrast to other papers in the literature on non-cooperative bargaining theory, we have adopted a more abstract approach to bargaining by referring to properties of bargaining procedures instead of specifying a precise protocol (game). In this respect, our analysis is more general than the standard framework in multilateral bargaining theory.

The games in our domain all have a common structure of sequential moves as is the case in the seminal models by Rubinstein (1982) and Selten (1981). But the crucial properties of bargaining games in our domain are summarized by:

1. every game is composed of a (possible infinite) number of sessions, each consisting of a finite number of agreement attempts,
2. the structure of each agreement attempt is rich enough to allow a significant flexibility in terms of the bargaining protocols, e.g.,

the order of moves, and the type of actions available to players (making proposals, making demands, responding, threatening, etc.).

One may suspect that for the purpose of equilibrium analysis of a typical game in our class a large number of the allowable moves are not substantive but simply ‘redundant’. We, however, think that scrutinizing redundant moves in bargaining procedures is not a trivial matter. The literature of bargaining theory often emphasizes the fact that the equilibrium analysis is very sensitive to the precise rules of the game. Moreover, the literature of cheap-talk games has indicated that payoff-irrelevant messages may affect the set of equilibrium outcomes. Indeed, one of the purposes of this paper is to address this issue of robustness by determining properties that set its limit. We have proved that our non-cooperative characterization of the core is robust with respect to the rules of the game as long as properties (A1)–(A4) hold.

Finally, we note that our analysis is restricted to mechanisms with a sequential structure of moves, which seems to us to be the natural framework in the context of bargaining. An alternative interesting project would be to establish similar results by referring to a class of games with simultaneous moves.

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NOTES

1. We remark that our results can be generalized to games with non-transferable utility (NTU-games) without much difficulty.

2. For the empty set ϕ, we set $x(\phi) = 0$.
3. We use the terminology of GAME FORM to distinguish from the standard definition of a game, in which terminal nodes are associated with payoffs.
4. We assume no other chance moves in our model, but an appropriate change in Property (A1) would allow for chance moves in other parts of the model.
5. The choice of responders in the proposal model and of demanders in the demand-commitment model is flexible enough to include the following rules : (i) a fixed ordering (seniority, numbering, etc.) on S and (ii) strategic selection by the preceding players.
6. We remark that an initiator in the first agreement attempt of every session may be chosen by a chance move. See condition (i) on an agreement attempt.
7. This notion is due to Harsanyi and Selten (1988).
8. See (A4) in Section 3.
9. Note that it is sufficient to impose Axiom 3 only on the behavior of players acting last in an agreement attempt as the proof of the theorem will show.

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