

# PROFINITE COMPLETIONS AND KAZHDAN'S PROPERTY (T)

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ABSTRACT. We show that property (T) is not profinite, that is, we construct two finitely generated residually finite groups which have isomorphic profinite completions while one admits property (T) and the other does not. This settles a question raised by M. Kassabov.

## 1. INTRODUCTION

Two finitely generated groups have isomorphic profinite completions if and only if they have the same collection of finite quotients. A property  $\mathcal{P}$  of finitely generated residually finite groups is called a *profinite property* if the following is satisfied: if  $\Gamma_1$  and  $\Gamma_2$  are such groups with  $\widehat{\Gamma}_1 \cong \widehat{\Gamma}_2$  (i.e., the profinite completion of  $\Gamma_1$  and  $\Gamma_2$  are isomorphic) then  $\Gamma_1$  has  $\mathcal{P}$  if and only if  $\Gamma_2$  has  $\mathcal{P}$ .

There are various interesting properties which are trivially profinite properties: e.g., having infinite Abelianization, rate of subgroup growth etc. But there are other profinite properties which are in some sense less trivial, for example, having polynomial word growth (this follows from Gromov's seminal result on polynomial growth) and for a finitely presented group the property of being large (a group is said to be *large* if one of its finite index subgroups has a free non-Abelian quotient)[Lac07]. On the other hand, various other properties are not profinite. In a recent work, it is shown in [CBW09] that there exist two finitely presented residually-finite groups such that one is conjugacy separable and the other is not, and yet they have isomorphic profinite completions; this shows that being conjugacy separable is not a profinite property. In a work in preparation, Kassabov showed that it follows from [Kas07],[KN06] that property  $(\tau)$  is not a profinite property, and he asked whether Kazhdan property (T) is a profinite property. In this note, we show that Kazhdan property (T) is not a profinite property. Explicitly we prove:

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**Theorem 1.** *Let  $D$  be a positive square-free integer,  $k := \mathbb{Q}(\sqrt{D})$  and  $\mathcal{O}_k$  its ring of integers. Fix an integer  $n > 7$  and let  $\Gamma = Spin(1, n)(\mathcal{O}_k)$  and  $\Lambda = Spin(5, n - 4)(\mathcal{O}_k)$ . Then, there exist finite-index subgroups  $\Gamma_0 < \Gamma$  and  $\Lambda_0 < \Lambda$  such that the profinite completion of  $\Gamma_0$  is isomorphic to the profinite completion of  $\Lambda_0$  while  $\Lambda_0$  admits property (T) and  $\Gamma_0$  does not. Therefore, Kazhdan property (T) is not profinite.*

In particular, there exist non-isomorphic arithmetic groups with isomorphic profinite completions. Note that  $Spin(1, n)(\mathcal{O}_k)$  (resp.  $Spin(5, n - 4)(\mathcal{O}_k)$ ) are central extensions of irreducible lattices in  $H_1 := SO(1, n) \times SO(1, n)$  (resp.  $H_2 := SO(5, n - 4) \times SO(5, n - 4)$ ). We can therefore deduce that  $H_1$  and  $H_2$  have lattices  $\Gamma_1$  and  $\Lambda_1$  resp. with isomorphic profinite completions. As  $\mathbb{R} - rank(H_1) = 2$  while  $\mathbb{R} - rank(H_2) = 10$  we see that the rank of the ambient Lie group is also not a profinite property (see also Remark 3.1 and compare with [PR72].)

While we can construct many non-trivial examples of non-isomorphic arithmetic groups with isomorphic profinite completions as the one above, we show in a work in preparation that every set of higher rank arithmetic groups with isomorphic profinite completion consists of *finitely many* isomorphism classes.

Our construction is based on a very simple idea. Vaguely speaking, the groups above have the congruence subgroup property and hence their profinite completions are essentially a product of the  $p$ -adic completions. Their  $p$ -adic completions agree since the quadratic forms  $(1, n)$  and  $(5, n - 4)$  agree in every local field. Nevertheless, they do not agree over  $\mathbb{R}$ . Moreover,  $\Gamma_0$  is a lattice in  $Spin(1, n) \times Spin(1, n)$  while  $\Lambda_0$  is a lattice in  $Spin(5, n - 4) \times Spin(5, n - 4)$ . The latter has (T) while the former does not. We also note that both groups admit property tau.

This note is organized as follows: Much of our construction relies on quadratic forms theory and their Clifford algebra and spin groups, so we review the relevant results and definitions in §2 together with a basic Lemma on profinite completions. In §3 we prove Theorem 1.

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## 2. PRELIMINARIES

**2.1. Quadratic forms and their spin groups.** We denote the ring of  $p$ -adic integers by  $\mathbb{Z}_p$ .

**Lemma 2.** *The quadratic forms*

$$q_1 := x_1^2 + x_2^2 + x_3^2 + x_4^2, \quad \text{and} \quad q_2 := -q_1 = -x_1^2 - x_2^2 - x_3^2 - x_4^2$$

are equivalent over  $\mathbb{Z}_p$  for every prime  $p$ , that is for each prime  $p$  there exist  $M_p \in GL_4(\mathbb{Z}_p)$  such that  $M_p M_p^t = -I$ .

*Proof.* For  $p \neq 2$  the Lemma follows directly from [Cas78, Ch.6 §1, Corollary to Theorem 1.3]. For  $p = 2$ , consider

$$M_2 := \begin{pmatrix} 2 & 1 & 1 & \alpha \\ 1 & -2 & \alpha & -1 \\ \alpha & 1 & -1 & -2 \\ 1 & -\alpha & -2 & 1 \end{pmatrix}$$

where  $\alpha^2 = -7, \alpha \in \mathbb{Z}_2$ . We have that  $M_2 \in M_4(\mathbb{Z}_2)$ ,  $M_2 M_2^t = -I$  and  $\det(M_2) = 1$ , i.e.,  $M_2$  shows that  $q_1$  and  $q_2$  are equivalent over  $\mathbb{Z}_2$ .  $\square$

**Corollary 3.** *Let  $q_{m,n} = \sum_{i=1}^m x_i^2 - \sum_{i=1}^n x_i^2$ . For  $m \geq 4$ , the quadratic form  $q_{m,n}$  and  $q_{m-4,n+4}$  are integrally equivalent over  $\mathbb{Z}_p$  for all primes  $p$ . It follows that for any number field  $k$  and a finite place  $v$ , we have that  $q_{m,n}$  and  $q_{m-4,n+4}$  are equivalent over  $\mathcal{O}_v$ , where  $\mathcal{O}_v$  is the ring of integers of  $k_v$ .*

*Proof.* Let  $V_i$  be the quadratic space associated to  $q_i, i = 1, 2$ , and  $V_{m,n}$  the quadratic space associated to  $q_{m,n}$ . Then for  $m \geq 4$ ,

$$V_{m,n} \cong V_1 \oplus V_{m-4,n} \quad \text{and} \quad V_{m-4,n+4} \cong V_2 \oplus V_{m-4,n}$$

as quadratic spaces. As  $V_1 \cong V_2$  over  $\mathbb{Z}_p$  for all  $p$ , so does  $V_{m,n}$  and  $V_{m-4,n+4}$ .  $\square$

In order to fix our notation and to fix specific representations of the groups involved, we recall the relevant definitions and give an explicit definition of the Clifford algebra and the spin group of a quadratic form. All objects and results that are described here can be found in [Cas78] (in particular, chapter 10 §2) or in [FH91, §20].

Given a non-degenerate quadratic space  $(V, \phi)$  of dimension  $n$  over a field  $k$ , there exists an algebra  $C(V, \phi)$  over  $k$  that contains  $V$  as a linear subspace and satisfies:

- (1)  $C(V, \phi)$  is of dimension  $2^n$  as a vector space.
- (2) For all  $x \in V$ ,  $x \cdot x = \phi(x)$ .
- (3)  $C(V, \phi)$  is generated as an algebra by  $V$ .

Moreover, these properties determine  $C(V, \phi)$  uniquely (up-to a  $k$ -algebra isomorphism that fix  $V$ ), and it is called the Clifford algebra of  $V$ .

We now give an explicit description of  $C(V, \phi)$ . This description depends on a choice of a normal basis, that is, a basis which is orthogonal with respect to the bilinear form associated to the quadratic form (see [Cas78, Chapter 10 §2]). Every non-degenerate quadratic space admits a normal basis and we fix a normal basis  $e_1, \dots, e_n$  for  $(V, \phi)$ . Recall that  $C(V, \phi)$  contains  $V$  and let  $J$  be any subset of  $\{1, 2, \dots, n\}$  arranged in ascending order, say

$$j_1 < j_2 < \dots < j_r$$

where  $r \leq n$ , and let

$$e(J) := e_{j_1} \cdot e_{j_2} \cdot \dots \cdot e_{j_r},$$

be the multiplication of the  $e_j$ 's and let  $e(\emptyset)$  be the unit element. Then, the set

$$\{e(J) : J \subset \{1, 2, \dots, n\} \text{ ordered in ascending order}, 0 \leq |J| \leq n\}$$

is a basis of  $C(V, \phi)$ . As  $C(V, \phi)$  is also generated by  $V$ , the multiplication in  $C(V, \phi)$  is determined by

$$e_i e_i = \phi(e_i)$$

and for  $i \neq j$ ,

$$e_i e_j = -e_j e_i.$$

The even Clifford algebra of  $C(V, \phi)$  is the algebra generated by  $\{e(J) : |J| \text{ is even}\}$ , and is denoted by  $C^0(V, \phi)$ . There exists an involution

$$' : C(V, \phi) \rightarrow C(V, \phi)$$

defined on the basis elements by  $(e(J))' = e_{j_r} \cdot e_{j_{r-1}} \cdot \dots \cdot e_{j_1}$  when  $J$  is as above, and extended linearly. Let  $(C^0(V, \phi))^*$  denote the group of invertible elements of  $C^0(V, \phi)$  and define  $Spin(V, \phi)$  as

$$Spin(V, \phi) := \{x \in (C^0(V, \phi))^* : xx' = 1, xVx' \subseteq V\}.$$

The group  $Spin(V, \phi)$  admits a faithful irreducible linear representation to  $GL(C^0(V, \phi))$ , by acting as right multiplication. We endow  $GL(C^0(V, \phi))$  with the structure that is induced from the basis  $\{e(J) : |J| \text{ is even}\}$  of  $C^0(V, \phi)$ . This gives a representation of  $Spin(V, \phi)$  to  $GL_{2^{n-1}}(\mathbb{C})$  and we identify  $Spin(V, \phi)$  with its image in this representation.

We now turn to the case of  $(V, \phi) = (V_{m,n}, q_{m,n})$ . Let

$$G_{m,n} = Spin(V_{m,n}, q_{m,n}) \quad \text{and} \quad C_{m,n} = C(V_{m,n}, q_{m,n}).$$

We choose the basis  $e_1, \dots, e_m, e_{m+1}, \dots, e_{m+n}$  of  $V_{m,n}$  that satisfy

$$(1) \quad q_{m,n}(e_i) = \begin{cases} 1 & \text{if } 1 \leq i \leq m \\ -1 & \text{if } m+1 \leq i \leq m+n \end{cases}$$

which is normal. Using this basis we get by the above construction a specific faithful irreducible representation of  $G_{m,n}$  to  $GL(C_{m,n}^0) \cong GL_{2m+n-1}(\mathbb{C})$ , by acting as right multiplication. The isomorphism  $GL(C_{m,n}^0) \cong GL_{2m+n-1}(\mathbb{C})$  depends on our specific choice of a normal basis and we fix this choice throughout. For any ring  $R \subset \mathbb{C}$ , we let  $G_{m,n}(R) := G_{m,n} \cap GL_{2m+n-1}(R)$  and call this group the  $R$ -points of  $G_{m,n}$ . One can show that the (conjugacy class of the) representation of  $G_{m,n}$  is independent of a choice of normal basis for the quadratic space, but the group of  $R$ -point for a general ring  $R$ , does depend on this choice.

The group  $G_{m,n}$  is known to be an almost simple and absolutely simple algebraic group defined over  $\mathbb{Q}$ . For any field  $k \subset \mathbb{R}$ ,  $\text{k-rank}(G_{m,n}) = \min(m, n)$  (See for example, [FH91, §20]). Moreover, the following follows from Corollary 3:

**Corollary 4.** *Let  $m > 4, n > 0$  be natural numbers,  $k$  be any number field and  $v$  a finite valuation on  $k$ . Then, under the fixed representations which are described above,  $G_{m,n}(\mathcal{O}_v)$  is isomorphic to  $G_{m-4, n+4}(\mathcal{O}_v)$ , where  $\mathcal{O}_v$  denotes the ring of integers in the completion of  $k$  with respect to  $v$ .*

*Proof.* Using Corollary 3, this follows readily from the definitions in [Cas78, Chapter 10 §2]. Since this is crucial for our construction, we give the main part of the argument.

Let  $m' = m - 4, n' = n + 4$ , and let  $k_v$  denote the completion of  $k$  with respect to  $v$ . Let  $(k_v^{m+n}, q_{m,n})$  be the quadratic space associated  $q_{m,n}$  with the standard normal basis  $e_1, \dots, e_{m+n}$ , that is,

$$(2) \quad q_{m,n}(e_i) = \begin{cases} 1 & \text{if } 1 \leq i \leq m \\ -1 & \text{if } m+1 \leq i \leq m+n. \end{cases}$$

By Corollary 3, there exists  $M \in GL_{m+n}(\mathbb{Z}_p)$  such that  $\{f_i := Me_i\}_{i=1}^{m+n}$  is also a normal basis of  $(k_v^{m+n}, q_{m,n})$  with

$$(3) \quad q_{m,n}(f_i) = \begin{cases} 1 & \text{if } 1 \leq i \leq m' \\ -1 & \text{if } m'+1 \leq i \leq m'+n'. \end{cases}$$

Let  $C := C^0(k_v^{m+n}, q_{m,n})$  and  $C' = C^0(k_v^{m'+n'}, q_{m',n'})$ . We show that  $GL(C)(\mathcal{O}_v) \cong GL(C')(\mathcal{O}_v)$  and the corollary readily follows as  $G_{m,n}(\mathcal{O}_v) := G_{m,n} \cap GL(C)(\mathcal{O}_v)$  (and resp. for  $G_{m',n'}$ ).

The construction above show that the bases  $\{e_i\}$  and  $\{f_i\}$  give rise to different bases on  $C$ , which we denote by  $E$  and  $F$ . The bases  $E$  and  $F$  gives rise to two structure on  $GL(C)$ , i.e., two isomorphisms  $\Phi_E, \Phi_F : GL(C) \rightarrow GL_{2m+n-1}(\mathbb{C})$ . The base  $E$  is the base fixed above and by  $GL(C)(\mathcal{O}_v)$  we mean the  $\mathcal{O}_v$ -points of  $GL(C)$  with respect to the basis  $E$ , i.e.,  $\Phi_E^{-1}(GL_{2m+n}(\mathcal{O}_v))$ . By equation 3 above,  $GL(C')(\mathcal{O}_v)$  is isomorphic to the  $\mathcal{O}_v$ -points of  $GL(C)$  with respect to the basis  $F$ . So we may

conclude by showing that the  $\mathcal{O}_v$ -points of  $GL(C)$  with respect to  $E$  are isomorphic to the  $\mathcal{O}_v$ -points of  $GL(C)$  with respect to  $F$ .

Let  $\tilde{M}$  denote the base change matrix from  $E$  to  $F$ , which is called the derived matrix of  $M$ . From the multiplication rules in  $C$  and the fact that  $M$  has entries in  $\mathbb{Z}_p$  and  $q_{m,n}$  and  $q_{m',n'}$  has coefficients in  $\mathbb{Z}_p$ , it follows that  $\tilde{M}$  also has entries in  $\mathbb{Z}_p$ . The inverse of  $\tilde{M}$  is the derived matrix of  $M^{-1}$ , which also has entries in  $\mathbb{Z}_p$  by the same argument. Conjugation by  $\tilde{M}$ , which we denote by  $Int(\tilde{M})$ , identify the representations  $\Phi_E, \Phi_F$ , that is,  $Int(\tilde{M}) \circ \Phi_E = \Phi_F$ . It follows that the  $\mathcal{O}_v$ -points of  $GL(C)$  with respect to  $E$  and with respect to  $F$  are isomorphic by conjugation by  $\tilde{M}$ .  $\square$

## 2.2. A basic Lemma on profinite completion.

**Lemma 5.** *Let  $\Gamma$  be a residually finite group. There is a one-to-one correspondence between the set  $\mathcal{X}$  of all finite-index subgroups of  $\Gamma$  and the set  $\mathcal{Y}$  of all open subgroup of  $\hat{\Gamma}$ , given by*

$$\begin{aligned} X &\mapsto \bar{X} \quad (X \in \mathcal{X}) \\ Y &\mapsto Y \cap \Gamma \quad (Y \in \mathcal{Y}) \end{aligned}$$

where  $\bar{X}$  denotes the closure of  $X$  in  $\hat{\Gamma}$ . Moreover,  $\bar{X}$  is canonically isomorphic to  $\hat{X}$  and

$$[\Gamma : X] = [\hat{\Gamma} : \hat{X}].$$

*Proof.* See [LS03, Proposition 16.4.3]  $\square$

## 3. PROPERTY (T) IN NOT PROFINITE - PROOF OF THEOREM 1

We continue with the notation of Theorem 1 and we let  $G_1 := G_{n,1}$  and  $G_2 := G_{n-4,5}$  with the fixed representations we described above. Let  $\sigma_1, \sigma_2$  be the two distinct embeddings of  $k$  into  $\mathbb{R}$ . They induce natural embeddings  $\hat{\sigma}_1, \hat{\sigma}_2$  of  $G_i(\mathcal{O}_k)$  to  $G_i(\mathbb{R})$ . We embed

$$\Gamma := G_1(\mathcal{O}_k) \rightarrow G_1(\mathbb{R}) \times G_1(\mathbb{R}), x \mapsto (\hat{\sigma}_1(x), \hat{\sigma}_2(x)),$$

and similarly

$$\Lambda := G_2(\mathcal{O}_k) \rightarrow G_2(\mathbb{R}) \times G_2(\mathbb{R}), x \mapsto (\hat{\sigma}_1(x), \hat{\sigma}_2(x))$$

It is well-known that these embeddings realize  $\Gamma$  and  $\Lambda$  as irreducible lattices.

We now show that any finite-index subgroup  $\Gamma_0 < \Gamma$  does not have property (T) while any finite-index subgroup  $\Lambda_0 < \Lambda$  does. The group  $G_1(\mathbb{R}) = Spin(n, 1)(\mathbb{R})$ , which is a central extension of  $SO(n, 1)$ , does not have property (T) ([BdlHV08, Theorem 3.5.4]) and neither does the direct product of  $G_1(\mathbb{R})$  with itself. ([BdlHV08, Proposition 1.7.8]). Note that any finite-index subgroup  $\Gamma_0 < \Gamma$  is a lattice in  $G_1(\mathbb{R}) \times G_1(\mathbb{R})$ . Since a

lattice in a group has property (T) if and only if the group has property (T) ([BdlHV08, Proposition 1.7.1]), it follows that any finite-index subgroup  $\Gamma_0 < \Gamma$  does not have property (T).

In contrast, the group  $G_2(\mathbb{R}) = Spin(n-4, 5)(\mathbb{R})$ , which is a central extension of  $SO(n-4, 5)$ , does have property (T) since it is almost simple of rank  $\geq 2$  and therefore so does its direct product with itself ([BdlHV08, Proposition 1.7.8]). Again, any finite-index subgroup  $\Lambda_0 < \Lambda$  is a lattice in  $G_2(\mathbb{R}) \times G_2(\mathbb{R})$ . It follows that any finite-index subgroup  $\Lambda_0 < \Lambda$  has property (T). In particular, a finite-index subgroup of  $\Gamma$  cannot be isomorphic to a finite-index subgroup of  $\Lambda$ .

Nevertheless, we will now show that there exist a finite-index subgroup  $\Gamma_0 < \Gamma$  and a finite-index subgroup  $\Lambda_0 < \Lambda$  that have isomorphic profinite completions. First note that by Corollary 4,  $G_1(\mathcal{O}_v) \cong G_2(\mathcal{O}_v)$  for any finite valuation  $v$ , so there exists an isomorphism

$$\Phi : \prod_v G_1(\mathcal{O}_v) \rightarrow \prod_v G_2(\mathcal{O}_v)$$

where the product runs over all the finite valuations of  $k$ .

By [Kne79, 11.3] the congruence kernel of  $G_2$  is the trivial group and therefore

$$\widehat{\Lambda} = \widehat{G_2(\mathcal{O}_k)} \cong G_2(\widehat{\mathcal{O}_k}) = \prod_v G_2(\mathcal{O}_v)$$

where the product runs over all discrete valuations of  $k$ . For  $G_1$ , by [Kne79, 11.5c] the congruence kernel is of size 1 or 2. In case the kernel is of size 1, it follows as above that  $\widehat{\Gamma}$  is isomorphic to

$$\prod_v G_1(\mathcal{O}_v),$$

and therefore  $\widehat{\Gamma}$  and  $\widehat{\Lambda}$  are isomorphic by  $\Phi$ . In case the kernel is of size 2,  $\widehat{\Gamma}$  fits into the following short exact sequence

$$1 \rightarrow \mathbb{Z}/2\mathbb{Z} \rightarrow \widehat{\Gamma} \rightarrow \prod_v G_1(\mathcal{O}_v) \rightarrow 1.$$

As  $\widehat{\Gamma}$  is profinite, we can find a finite-index open subgroup that intersects  $\mathbb{Z}/2\mathbb{Z}$  trivially and therefore maps to  $\prod_v G_1(\mathcal{O}_v)$  injectively. By Lemma 5, this subgroup is necessarily of the form  $\widehat{\Gamma}_0$ , for a finite-index subgroup  $\Gamma_0$  of  $\Gamma$ . We note that the profinite completion of an arithmetic subgroup intersect  $\mathbb{Z}/2\mathbb{Z}$  non-trivially if and only if it is a congruence subgroup, so any non-congruence arithmetic subgroup can be chosen for  $\Gamma_0$ .

We identify  $\widehat{\Gamma}_0$  with its (faithful) image in  $\prod_v G_1(\mathcal{O}_v)$ . By Lemma 5 there exists a finite-index subgroup  $\Lambda_0$  of  $\Lambda$  with  $\widehat{\Lambda}_0 \cong \Phi(\widehat{\Gamma}_0)$ . Thus  $\widehat{\Lambda}_0$  and  $\widehat{\Gamma}_0$  are isomorphic. As explained above,  $\Lambda_0$  has property (T) and  $\Gamma_0$  does not. This concludes the proof of Theorem 1.

**3.1. Remark.** The above proof also show that the rank of the Lie group containing  $\Gamma$  as a lattice is not a profinite property. Indeed, using Corollary 4 and induction, one sees that for arbitrarily large fixed  $n$ , and every  $0 \leq k \leq \frac{n}{8}$  the groups  $G_k \times G_k$  with  $G_k := Spin(4k+1, n-4k)$  have irreducible lattice  $\Gamma_k$  which share the same profinite completions, but are pairwise non-isomorphic, by Mostow's strong-rigidty (which may be applied to the images of  $\Gamma_k$  in  $SO(4k+1, n-4k) \times SO(4k+1, n-4k)$ ). The rank of  $G_k \times G_k$  is  $2(4k+1)$ .

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