Dimension reduction and other topics in discrete metric geometry

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In 1981 Lindenstrauss and I proved what has come to be known as the J-L lemma: if A is any set of n points in a Euclidean space, then A can be realized, with constant distortion, in $\mathbb{R}^d$ with $d \leq 1 + \log n$. This means that there is a function F from A into $\mathbb{R}^d$ so that for every pair of points x and y in A, $|x - y| \leq |F(x) - F(y)| \leq C|x - y|$. Moreover, F can be taken to be the restriction of a linear mapping from the span of A into $\mathbb{R}^d$ (this stronger version is called the linear J-L lemma). I'll review the proof of this old lemma and mention the application of it in the original paper Joram and I wrote, and then discuss more recent results on dimension reduction, including

1. The theorem of Brinkman and Charikar that the J-L lemma fails in L_1. (The proof I'll outline is an argument due to Schechtman and me that further simplifies the beautiful argument of Lee and Naor.)

2. The result of Naor and mine that while there are spaces other than Hilbert spaces that satisfy the linear J-L lemma, any Banach space that satisfies the lemma must, in a certain sense, be extremely close to a Hilbert space.

If time permits, I will discuss a direction in metric geometry that I believe should be fruitful; namely, a theory of operator ideals for Lipschitz mappings from a metric spaces into normed spaces that mimics the well developed theory of operator ideals for linear mappings between normed spaces.