Homework #4

Exercise 1.
Prove that if a tensor is symmetric (antisymmetric) in some basis it is symmetric (antisymmetric) in every basis.

Exercise 2.
Prove that an antisymmetric tensor of type $(p, 0)$ or $(0, p)$ with $p > n$ is zero $(dimV = n)$.

Exercise 3.
Prove that

$$dim\Lambda^p(V) = \frac{n!}{p!(n-p)!}$$

Exercise 4.
Prove that

$$\begin{array}{ll} 1) & (\alpha \wedge \beta) \wedge \gamma = \alpha \wedge (\beta \wedge \gamma) \\ 2) & (\alpha + \beta) \wedge \gamma = \alpha \wedge \gamma + \beta \wedge \gamma \\ 3) & k(\alpha \wedge \beta) = (k\alpha) \wedge \beta = \alpha \wedge (k\beta) \\ 4) & \alpha \wedge \beta = (-1)^{pq} \beta \wedge \gamma \end{array}$$

Exercise 5.
Prove that

$$\begin{array}{ll} 1) & v\rfloor(\alpha + \beta) = v\rfloor\alpha + v\rfloor\beta \\ 2) & (v + u)\rfloor\alpha = v\rfloor\alpha + u\rfloor\alpha \\ 3) & v\rfloor u\rfloor w = -u\rfloor v\rfloor w \\ 4) & v\rfloor(\alpha \wedge \beta) = (v\rfloor\alpha) \wedge \beta + (-1)^{\deg \alpha} \alpha \wedge (v\rfloor\beta) \end{array}$$

Exercise 6.
Prove that for any p-form w

$$\vartheta^a \wedge (e_a\rfloor w) = pw$$

$$e_a\rfloor(\vartheta^a \wedge w) = (n-p)w$$