

Homework #2

**Exercise 1.**

Find *all* linear transformations of  $t, x$  that preserve the form

$$\phi_{tt} - \phi_{xx}$$

**Exercise 2.**

Check the triangle inequality, the parallelogram identity and Schwarz's inequality for Lorentzian vectors.

In the following questions, use Schwarz's inequality for Euclidean vectors

$$|u_i v^i| \leq \sqrt{u^i u_i v^j v_j}$$

**Exercise 3.**

Prove, that two timelike vectors cannot be orthogonal.

**Exercise 4.**

Prove, that, for two timelike vectors  $u$  and  $v$  such that  $u^0 > 0, v^0 > 0$ , we have  $(u, v) < 0$ .

**Exercise 5.**

Prove, that a timelike vector cannot be orthogonal to a nonzero null vector.

**Exercise 6.**

Prove, that two nonzero null vectors are orthogonal if and only if they are scalar multiples of each other.