Homework #2

Exercise 1.
Prove the Lemma used in the derivative of the Euler-Lagrange equation.
If a continuous function $f(t)$ satisfies

$$\int_{t_1}^{t_2} f(t)h(t)dt = 0$$

for any continuous function $h(t)$ with $h(t_1) = h(t_2) = 0$, then $f(t) = 0$.

Exercise 2.
Check how change the action functional, the equation of motion and the energy under the following changes of the Lagrangian

$$L = f(\tilde{L}) \,, \qquad L = C_1\tilde{L} + C_2$$
$$L = \tilde{L} + \frac{d}{dt}\varphi(x, \dot{x}, t)$$

Exercise 3.
Derive the equation of motion and the energy for the system of two mass points with the Lagrangian

$$L = \frac{m_1\dot{x}^2}{2} + \frac{m_2\dot{y}^2}{2} - \frac{km_1m_2}{|x-y|}$$

Exercise 4.
Prove the identities

$$\mathrm{rot}(\mathrm{grad}\,\varphi) = 0$$
$$\mathrm{div}(\mathrm{rot}\,A) = 0$$
$$\mathrm{rot}(\mathrm{rot}\,A) = \mathrm{grad}(\mathrm{div}\,A) - \triangle A$$

Exercise 5.
Prove that

$$\mathrm{rot} A = 0 \qquad <===> \qquad A = \mathrm{grad}\,\varphi$$