Homework #1

Exercise 1.
Show that if a mechanical system consists of only one body, then its acceleration is equal to zero. ("Newton's first law").

Exercise 2.
Show that if a mechanical system consists of only two bodies, then whenever one body exerts force upon a second body, the second body exerts an equal and opposite force upon the first body ("Newton's third law").

Exercise 3.
Check the formulas

$$\varepsilon_{abc}\varepsilon^{kmn} = \det\begin{pmatrix} \delta^k_a & \delta^k_b & \delta^k_c \\ \delta^m_a & \delta^m_b & \delta^m_c \\ \delta^n_a & \delta^n_b & \delta^n_c \end{pmatrix}$$

$$\varepsilon_{abc}\varepsilon^{kmc} = \delta^k_a\delta^m_b - \delta^k_b\delta^m_a$$

$$\varepsilon_{abc}\varepsilon^{kbc} = 2\delta^k_a$$

$$\varepsilon_{abc}\varepsilon^{abc} = 6$$

Exercise 4.
Prove a formula for determinant of a matrix $h = \{h_i{}^j, i, j = 1, \cdots, n\}$

$$\det(h) = \frac{1}{n!}\varepsilon^{i_1\cdots i_n}\varepsilon_{j_1\cdots j_n}h_{i_1}{}^{j_1}\cdots h_{i_n}{}^{j_n}$$

Exercise 5.
Prove (no summation in j !)

$$\sum_{k=1}^{n} h^j{}_{i_k}\varepsilon_{j_1\cdots j_{k-1}jj_{k+1}\cdots j_n} = \varepsilon_{j_1\cdots j_n}\mathrm{Tr}\left(h^j{}_{i_k}\right)$$