Yisrael Aumann’s Science

Sergiu Hart

Israel Academy of Sciences and Humanities

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Aumann’s Short CV

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- **1998-2003:** Founding President of the Game Theory Society
2005
Recurrent games
Major Contributions

- Repeated games
- Perfect competition
Major Contributions

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- Correlated equilibrium
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- Interactive epistemology
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Repeated Games

“The theory of repeated games ...”

Aumann 1981, 1985
“The theory of *repeated games* ... Its aim is to account for phenomena such as *cooperation, altruism, revenge, threats* (self-destructive or otherwise) ... *concealing and revealing information* ... etc.

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Repeated Games

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Repeated Games

“The theory of *repeated games* ... Its aim is to account for phenomena such as cooperation, altruism, revenge, threats (self-destructive or otherwise) ... concealing and revealing information ... etc.—phenomena which may at first seem irrational—in *terms of the usual ‘selfish’ utility-maximizing paradigm* of game theory and neoclassical economics.”

*Aumann 1981, 1985*
Noncooperative strategic behavior in the repeated game yields Cooperative behavior.
Repeated Games

The Folk Theorem
Repeated Games

- The Folk Theorem
- The Strong Folk Theorem
  (*Aumann 1959*)
Repeated Games

- The Folk Theorem
- The Strong Folk Theorem
  (Aumann 1959)
- The Perfect Folk Theorem
Repeated Games

- The Folk Theorem
- The Strong Folk Theorem
  (Aumann 1959)
- The Perfect Folk Theorem
- Repeated Games of Incomplete Information
  (Aumann & Maschler 1966, Aumann, Maschler & Stearns 1968)
The Market

Pieter Bruegel the Elder (1559)
Perfect Competition

How should perfect competition be modelled?
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“... the influence of an individual participant on the economy cannot be mathematically negligible, as long as there are only finitely many participants.

Aumann 1964
How should **perfect competition** be modelled?

“... the influence of an individual participant on the economy **cannot be** mathematically **negligible**, as long as there are **only finitely many participants**.

... We submit that the most natural model for this purpose contains a **continuum** of participants, similar to the continuum of points on a line or the continuum of particles in a fluid.”

*Aumann 1964*
The Equivalence Principle

In *markets with a continuum of traders*:
The Equivalence Principle

In markets with a continuum of traders:

The set of Walrasian equilibria coincides with

the solutions of the corresponding “cooperative” game
The Equivalence Principle

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(core, value, ...)

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The Equivalence Principle

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The set of *Walrasian equilibria*

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A Correlated Equilibrium is a Nash equilibrium when the players receive payoff-irrelevant signals before playing the game.
Interactive Epistemology

- **Formal model** of knowledge, knowledge about knowledge, and common knowledge

  *Aumann 1976, 1999ab*
Interactive Epistemology

- **Formal model** of knowledge, knowledge about knowledge, and common knowledge

**The Agreement Theorem:**

*If two people have the same prior, and their posteriors for an event $A$ are common knowledge, then their posteriors must be equal.*

Aumann 1976, 1999ab
Assume a common prior.
Assume a common prior.

If all players are rational,
Assume a common prior.

If all players are rational, and this is common knowledge,
Rationality

Assume a common prior.

If all players are rational, and this is common knowledge,

then

their play constitutes a correlated equilibrium

Aumann 1987
Other Major Contributions

- Cooperative games (NTU, core, value, bargaining set, ...)

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- Subjective probability and utility
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Power and taxes
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- Coalitions
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“Unlike other approaches to disciplines like economics or political science, GAME THEORY does not use different, ad-hoc constructs to deal with various specific issues, such as perfect competition, monopoly, oligopoly, international trade, taxation, voting, deterrence, animal behavior, and so on.

Aumann’s interview 2004
“Unlike other approaches to disciplines like economics or political science, GAME THEORY does not use different, ad-hoc constructs to deal with various specific issues, such as perfect competition, monopoly, oligopoly, international trade, taxation, voting, deterrence, animal behavior, and so on. Rather, it develops methodologies that apply in principle to all interactive situations, then sees where these methodologies lead in each specific application.”

Aumann’s interview 2004
Aumann’s Doctoral Students

1. Bezalel Peleg
2. David Schmeidler
3. Shmuel Zamir
4. Elon Kohlberg
5. Benyamin Shitovitz
6. Zvi Artstein
7. Eugene Wesley
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<tr>
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<td>1</td>
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<td>9</td>
<td>Abraham Neyman</td>
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<td>10</td>
<td>Yair Tauman</td>
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<td>Yossi Feinberg</td>
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S. Hart, Introduction to Aumann’s interview 2004
“A pure mathematician who is a renowned economist, Aumann has been a central figure in developing game theory and establishing its key role in modern economics. He has shaped the field through his fundamental and pioneering work, work that is conceptually profound, and much of it also mathematically deep.”

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A Scientist at Work