Two(!) Good To Be True

Sergiu Hart

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האוניברסיטה העברית בירושלים
Two(!) Good To Be True

Sergiu Hart
Center for the Study of Rationality
Dept of Economics    Dept of Mathematics
The Hebrew University of Jerusalem

hart@huji.ac.il
http://www.ma.huji.ac.il/hart
Sergiu Hart and Phil Reny
“Revenue Maximization in Two Dimensions”
(2010, in preparation)
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“Revenue Maximization in Two Dimensions”
(2010, in preparation)

Sergiu Hart and Phil Reny
“Maximizing Revenue with Multiple Goods: Nonmonotonicity and Other Observations”
(2011)

www.ma.huji.ac.il/hart/abs/monot-m.html
Sergiu Hart and Phil Reny
“Implementation of Reduced Form Mechanisms: A Simple Approach and a New Characterization” (2011)

www.ma.huji.ac.il/hart/abs/q-mech.html
Sergiu Hart and Noam Nisan
“Approximate Revenue Maximization with Multiple Items”
(2012)
www.ma.huji.ac.il/hart/abs/m-approx.html
Sergiu Hart and Noam Nisan
“Approximate Revenue Maximization with Multiple Items”
(2012)
www.ma.huji.ac.il/hart/abs/m-approx.html

Sergiu Hart and Noam Nisan
“The Menu-Size Complexity of Auctions”
(2013)
www.ma.huji.ac.il/hart/abs/m-corr.html
A Simple Problem
A Simple Problem

1 SELLER
A Simple Problem

1 SELLER

1 BUYER
A Simple Problem

- 1 SELLER
- 1 BUYER
- $k$ GOODS (ITEMS)
A Simple Problem

- 1 SELLER
- 1 BUYER
- $k$ GOODS (ITEMS)

**OBJECTIVE:**

MAXIMIZE the REVENUE of the SELLER
A Simple Problem

1 SELLER

1 BUYER

k GOODS (ITEMS)
A Simple Problem

- **1 SELLER**
- **1 BUYER**
- **$k$ GOODS (ITEMS)**
- **values of GOODS to BUYER:**
  \[ x = (x_1, x_2, \ldots, x_k) \]
A Simple Problem

- 1 SELLER
- 1 BUYER
- $k$ GOODS (ITEMS)
  - values of GOODS to BUYER: $x = (x_1, x_2, \ldots, x_k)$
  - additive valuation
    (good 1 and good 2 = $x_1 + x_2$)
A Simple Problem

- **1 SELLER**
- **1 BUYER**
- **$k$ GOODS (ITEMS)**
  - values of GOODS to BUYER:
    \[ x = (x_1, x_2, \ldots, x_k) \]
  - additive valuation
    (good 1 and good 2 = $x_1 + x_2$)
  - **BUYER knows** the value $x$
A Simple Problem

- **1 SELLER**
- **1 BUYER**
- **k GOODS (ITEMS)**
  - values of GOODS to BUYER:
    \[ x = (x_1, x_2, ..., x_k) \]
  - additive valuation
    (good 1 and good 2 = \( x_1 + x_2 \))
  - **BUYER knows** the value \( x \)
  - **SELLER does not know** the value \( x \)
A Simple Problem

- 1 SELLER
- 1 BUYER
- \( k \) GOODS (ITEMS)
  - values of GOODS to BUYER:
    \[ x = (x_1, x_2, \ldots, x_k) \]
  - additive valuation
    (good 1 and good 2 = \( x_1 + x_2 \))
  - BUYER knows the value \( x \)
  - SELLER does not know the value \( x \)
  - SELLER knows the distribution \( F \) of \( x \)
A Simple Problem

1 SELLER

1 BUYER

$k$ GOODS (ITEMS)

values of GOODS to BUYER:
\[ x = (x_1, x_2, \ldots, x_k) \]

additive valuation
(good 1 and good 2 = \( x_1 + x_2 \))

BUYER knows the value \( x \)

SELLER does not know the value \( x \)

SELLER knows the distribution \( F \) of \( x \)
(\( F \) is a c.d.f. on \( \mathbb{R}^k_+ \))
A Simple Problem

1 SELLER

1 BUYER

\( k \) GOODS (ITEMS)
A Simple Problem

1 SELLER

1 BUYER

$k$ GOODS (ITEMS)

SELLER and BUYER:

quasi-linear utilities (i.e., additive in monetary payments)
A Simple Problem

- 1 SELLER
- 1 BUYER
- $k$ GOODS (ITEMS)

SELLER and BUYER:

- **quasi-linear** utilities (i.e., additive in monetary payments)
- **risk-neutral** (i.e., linear in probabilities)
A Simple Problem

- 1 SELLER
- 1 BUYER
- \( k \) GOODS (ITEMS)

SELLER and BUYER:
- **quasi-linear** utilities (i.e., additive in monetary payments)
- **risk-neutral** (i.e., linear in probabilities)
  (or: linear in quantities)
A Simple Problem

- 1 SELLER
- 1 BUYER
- \( k \) GOODS (ITEMS)

SELLER and BUYER:
- quasi-linear utilities (i.e., additive in monetary payments)
- risk-neutral (i.e., linear in probabilities)
  (or: linear in quantities)

SELLER:
- no value and no cost for the GOODS
A Simple Problem

- 1 SELLER
- 1 BUYER
- $k$ GOODS (ITEMS)
A Simple Problem

- 1 SELLER
- 1 BUYER
- $k$ GOODS (ITEMS)

**OBJECTIVE:**

MAXIMIZE the **REVENUE** of the **SELLER**
ONE GOOD ($k = 1$):
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- SELLER posts a PRICE $p$
ONE GOOD ($k = 1$):

- **SELLER** posts a **PRICE** $p$
- **BUYER** chooses between:
  - get the good and pay $p$, or
  - get nothing and pay nothing

---

Myerson 1981
ONE GOOD \((k = 1)\):

- **SELLER** posts a **PRICE** \(p\)
- **BUYER** chooses between:
  - get the good and pay \(p\), or
  - get nothing and pay nothing

\(p\) such that **REVENUE** \(R = p \cdot \Pr[X > p]\) is MAXIMAL

\[= p \cdot (1 - F(p))\]

Myerson 1981
One Good: Solution

One Good ($k = 1$):

- Seller posts a price $p$
- Buyer chooses between:
  - get the good and pay $p$, or
  - get nothing and pay nothing

- $p$ such that Revenue $R = p \cdot \Pr[X > p]$
  $= p \cdot (1 - F(p))$ is maximal

\[ \text{Rev}(F) = \max_p p \cdot (1 - F(p)) \]

Myerson 1981
One Good: Example
One Good: Example

\[ X \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
22 & \text{with probability } 1/2
\end{cases} \]
One Good: Example

\[ X \sim \begin{cases} 10 & \text{with probability } 1/2 \\ 22 & \text{with probability } 1/2 \end{cases} \]

\[ p = 10 \rightarrow R = 10 \cdot 1 = 10 \]
One Good: Example

\[ X \sim \begin{cases} 
10 & \text{with probability } \frac{1}{2} \\
22 & \text{with probability } \frac{1}{2} 
\end{cases} \]

\[ p = 10 \rightarrow R = 10 \cdot 1 = 10 \]

\[ p = 22 \rightarrow R = 22 \cdot \frac{1}{2} = 11 \]
One Good: Example

\[ X \sim \begin{cases} 
10 & \text{with probability } \frac{1}{2} \\
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\end{cases} \]

- \( p = 10 \rightarrow R = 10 \cdot 1 = 10 \)
- \( p = 22 \rightarrow R = 22 \cdot \frac{1}{2} = 11 \)
One Good: Example

\[ X \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
22 & \text{with probability } 1/2 
\end{cases} \]

- \( p = 10 \rightarrow R = 10 \cdot 1 = 10 \)
- \( p = 22 \rightarrow R = 22 \cdot 1/2 = 11 \)

\[ \text{REV}(F) = 11 \quad p = 22 \]
Two Goods

Two Goods \((k = 2)\)
Two Goods \((k = 2)\), Independent
Two Goods (\(k = 2\)), Independent

- post \(\text{PRICE } p_1\) for good 1
- post \(\text{PRICE } p_2\) for good 2
Two Goods

Two Goods \((k = 2)\), Independent
Two Goods: Example

Two Goods \((k = 2)\), Independent
Two Goods: Example

Two Goods \((k = 2)\), Independent

\[ X_1, X_2 \sim \begin{cases} 
10 & \text{with probability } \frac{1}{2} \\
22 & \text{with probability } \frac{1}{2}
\end{cases} \]
Two Goods: Example

Two Goods \((k = 2)\), Independent

\[ X_1, X_2 \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
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\end{cases} \]

\[ \text{Rev}(X_1) = \text{Rev}(X_2) = 11 \]
Two Goods: Example

Two Goods \((k = 2)\), Independent

\[ X_1, X_2 \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
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\end{cases} \]

\[ \text{REV}(X_1) = \text{REV}(X_2) = 11 \]

\[ \text{REV}(X_1) + \text{REV}(X_2) = 11 + 11 = 22 \]
Two Goods: Example

Two Goods \((k = 2)\), Independent

\[ X_1, X_2 \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
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\end{cases} \]

\[ \text{Rev}(X_1) = \text{Rev}(X_2) = 11 \]

\[ \text{Rev}(X_1) + \text{Rev}(X_2) = 11 + 11 = 22 \]

sell the two goods together ("bundle") for the price \(p_{12} = 32\) :
Two Goods Example

Two Goods \((k = 2)\), Independent

\[ X_1, X_2 \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
22 & \text{with probability } 1/2 
\end{cases} \]

- \( \text{Rev}(X_1) = \text{Rev}(X_2) = 11 \)
- \( \text{Rev}(X_1) + \text{Rev}(X_2) = 11 + 11 = 22 \)
- sell the two goods together ("bundle") for the price \( p_{12} = 32 : \)

\[ R = 32 \cdot \frac{3}{4} = 24 > 22 \]
Two Goods: Example

Two Goods ($k = 2$), Independent

\[ X_1, X_2 \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
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- \( \text{REV}(X_1) = \text{REV}(X_2) = 11 \)
- \( \text{REV}(X_1) + \text{REV}(X_2) = 11 + 11 = 22 \)

sell the two goods together ("bundle") for the price \( p_{12} = 32 \) :

\[ R = 32 \cdot \frac{3}{4} = \boxed{24} > 22 \]
General Mechanism

**OUTCOME** when **BUYER**’s valuation is $x = (x_1, x_2, ..., x_k)$:
General Mechanism

**OUTCOME** when **BUYER**’s valuation is $x = (x_1, x_2, \ldots, x_k)$:

$q_i(x) = \text{probability that **BUYER** gets good } i$
OUTCOME when BUYER’s valuation is $x = (x_1, x_2, \ldots, x_k)$:

$q_i(x) =$ probability that BUYER gets good $i$
(or: fractional quantity of good $i$)
**General Mechanism**

**OUTCOME** when **BUYER**’s valuation is \( x = (x_1, x_2, \ldots, x_k) \):

- \( q_i(x) = \text{probability} \) that **BUYER** gets good \( i \)  
  (or: fractional quantity of good \( i \))

- \( s(x) = \text{payment} \) from **BUYER** to **SELLER**  
  ("REVENUE")
General Mechanism

**OUTCOME** when **BUYER**’s valuation is \( x = (x_1, x_2, \ldots, x_k) \):

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- \( s(x) = \text{payment} \) from **BUYER** to **SELLER** ("REVENUE")

**BUYER**’s payoff (utility):
General Mechanism

**Outcome** when BUYER's valuation is \( x = (x_1, x_2, \ldots, x_k) \):

- \( q_i(x) = \text{probability that BUYER gets good } i \)
  (or: fractional quantity of good \( i \))

- \( s(x) = \text{payment from BUYER to SELLER} \)
  ("REVENUE")

**BUYER's payoff (utility):**

- \( b(x) = q_1(x) \cdot x_1 + \ldots + q_k(x) \cdot x_k - s(x) \)
Simple Mechanism

MENU: a SET of possible OUTCOMES
Simple Mechanism

**MENU**: a **SET** of possible **OUTCOMES**

- **SELLER** posts a **MENU**
Simple Mechanism

**MENU**: a **SET** of possible **OUTCOMES**

- **SELLER** posts a **MENU**
- **BUYER** chooses one **OUTCOME** in the **MENU**
Simple Mechanism

**MENU**: a **SET** of possible **OUTCOMES**

- **SELLER** posts a **MENU**
- **BUYER** chooses one **OUTCOME** in the **MENU**

\[ ((q_1(x), \ldots, q_k(x)), s(x)) = \text{the OUTCOME chosen by BUYER when valuation is } x \]
"Menu" Mechanism

MENU: a **SET** of possible OUTCOMES

- **SELLER** posts a MENU
- **BUYER** chooses one OUTCOME in the MENU

\[
((q_1(x), \ldots, q_k(x)), s(x)) = \text{the OUTCOME chosen by **BUYER** when valuation is } x
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"Menu" Mechanism

MENU: a SET of possible OUTCOMES

- SELLER posts a MENU
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\[
((q_1(x), \ldots, q_k(x)), s(x)) = \text{the OUTCOME chosen by BUYER when valuation is } x
\]

"The Revelation Principle":
Every mechanism is equivalent to a MENU MECHANISM
"Menu" Mechanism

**MENU**: a **SET** of possible **OUTCOMES**
- **SELLER** posts a **MENU**
- **BUYER** chooses one **OUTCOME** in the **MENU**

\[(q_1(x), ..., q_k(x)), s(x)\] = the **OUTCOME** chosen by **BUYER** when valuation is \(x\)

"**THE REVELATION PRINCIPLE"**: Every mechanism is equivalent to a **MENU MECHANISM** ("direct mechanism")
Buyer

- Incentive Compatibility (IC)
**Incentive Compatibility (IC)**

\[ q_1(x) \cdot x_1 + \ldots + q_k(x) \cdot x_k - s(x) \geq \]
\[ q_1(y) \cdot x_1 + \ldots + q_k(y) \cdot x_k - s(y) \]

(for all \( x \) and \( y \))
• Incentive Compatibility (IC)

\[ q_1(x) \cdot x_1 + \ldots + q_k(x) \cdot x_k - s(x) \geq q_1(y) \cdot x_1 + \ldots + q_k(y) \cdot x_k - s(y) \]

(for all \( x \) and \( y \))

• Individual Rationality (IR) / Participation
Incentive Compatibility (IC)

\[ q_1(x) \cdot x_1 + \ldots + q_k(x) \cdot x_k - s(x) \geq 0 \]
\[ q_1(y) \cdot x_1 + \ldots + q_k(y) \cdot x_k - s(y) \]

(for all \( x \) and \( y \))

Individual Rationality (IR) / Participation

\[ q_1(x) \cdot x_1 + \ldots + q_k(x) \cdot x_k - s(x) \geq 0 \]

(for all \( x \))
Maximize Revenue:
Maximize Revenue:

maximize

\[ R = \mathbb{E}_{x \sim F}[s(x)] = \int s(x) dF(x) \]
Maximize Revenue:

maximize

\[ R = \mathbb{E}_{x \sim F}[s(x)] = \int s(x) dF(x) \]

subject to

\((q, s)\) satisfies IC & IR
The Mathematical Problem
The Mathematical Problem

The problem of **MAXIMIZING REVENUE**: 

The problem of **MAXIMIZING REVENUE:**
The Mathematical Problem

The problem of **MAXIMIZING REVENUE**:

Reduces to maximizing a **linear function** over a **convex set** of functions.
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Reduces to maximizing a **linear function** over a **convex set** of functions
The Mathematical Problem

The problem of **MAXIMIZING REVENUE**: 

- Reduces to maximizing a **linear function** over a **convex set** of functions
- Is extremely difficult for 2 or more goods
The problem of **MAXIMIZING REVENUE**: 

- Reduces to maximizing a **linear function** over a **convex set** of functions
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The problem of **MAXIMIZING REVENUE**: 

- Reduces to maximizing a **linear function** over a **convex set** of functions
- Is extremely difficult for 2 or more goods

There is no (useful) **characterization** of the **extreme points** for dimensions $\geq 2$
Two Goods: Example 1
Two Goods: Example 1

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
22 & \text{with probability } 1/2 
\end{cases} \]
Two Goods: Example 1

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } \frac{1}{2} \\
22 & \text{with probability } \frac{1}{2}
\end{cases} \]

Separate:

\[ \text{REV}(X) + \text{REV}(Y) \]
Two Goods: Example 1

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
22 & \text{with probability } 1/2 
\end{cases} \]

Separate:
\[ \text{REV}(X) + \text{REV}(Y) = 11 + 11 = 22 \]
Two Goods: Example 1

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
22 & \text{with probability } 1/2
\end{cases} \]

- Separate:
  \[ \text{REV}(X) + \text{REV}(Y) = 11 + 11 = 22 \]

- Bundled:
  \[ \text{REV}(X + Y) \]
Two Goods: Example 1

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
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\end{cases} \]

- **Separate:**
  \[ \text{REV}(X) + \text{REV}(Y) = 11 + 11 = 22 \]

- **Bundled:**
  \[ \text{REV}(X + Y) = 32 \cdot 3/4 = 24 \]
Two Goods: Example 1

Independent and Identically Distributed (I.I.D.)

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Two Goods: Example 1

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 10 & \text{with probability } 1/2 \\ 22 & \text{with probability } 1/2 \end{cases} \]

- Separate:
  \[ \text{REV}(X) + \text{REV}(Y) = 11 + 11 = 22 \]

- Bundled:
  \[ \text{REV}(X + Y) = 32 \cdot 3/4 = 24 \]

PRICE FOR THE BUNDLE
Two Goods: Example 2

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } \frac{1}{2} \\
50 & \text{with probability } \frac{1}{2}
\end{cases} \]
Two Goods: Example 2

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 10 & \text{with probability } 1/2 \\ 50 & \text{with probability } 1/2 \end{cases} \]

Separate:

\[ \text{Rev}(X) + \text{Rev}(Y) \]
Two Goods: Example 2

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } \frac{1}{2} \\
50 & \text{with probability } \frac{1}{2}
\end{cases} \]

- Separate:
  \[ \text{Rev}(X) + \text{Rev}(Y) = 25 + 25 = 50 \]
Two Goods: Example 2

Independent and Identically Distributed (I.I.D.)

\( X, Y \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
50 & \text{with probability } 1/2 
\end{cases} \)

- **Separate:**
  \[
  \text{Rev}(X) + \text{Rev}(Y) = 25 + 25 = 50
  \]

- **Bundled:**
  \[
  \text{Rev}(X + Y) = 60 \cdot \frac{3}{4} = 45
  \]
Two Goods: Example 2

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
50 & \text{with probability } 1/2 
\end{cases} \]

- **Separate:**
  \[ \text{REV}(X) + \text{REV}(Y) = 25 + 25 = 50 \]

- **Bundled:**
  \[ \text{REV}(X + Y) = 60 \cdot 3/4 = 45 \]
Two Goods: Example 2

Independent and Identically Distributed (I.I.D.)

\[ X, Y \sim \begin{cases} 
10 & \text{with probability } 1/2 \\
50 & \text{with probability } 1/2 
\end{cases} \]

- **Separate:**
  \[ \text{REV}(X) + \text{REV}(Y) = 25 + 25 = 50 \]

- **Bundled:**
  \[ \text{REV}(X + Y) = 60 \cdot \frac{3}{4} = 45 \]

PRICE FOR EACH GOOD
Two Goods: Example 3

\[
X, Y \sim \begin{cases} 
0 & \text{w/probability } 1/3 \\
1 & \text{w/probability } 1/3 \\
2 & \text{w/probability } 1/3 \\
\end{cases} \quad \text{(I.I.D.)}
\]
Two Goods: Example 3

\[ X, Y \sim \begin{cases} 
0 \quad \text{w/probability } 1/3 \\
1 \quad \text{w/probability } 1/3 \\
2 \quad \text{w/probability } 1/3 
\end{cases} \quad \text{(I.I.D.)} \\
\]

Separate:
Two Goods: Example 3

\[ X, Y \sim \begin{cases} 
0 & \text{w/probability } \frac{1}{3} \\
1 & \text{w/probability } \frac{1}{3} \quad \text{(I.I.D.)} \\
2 & \text{w/probability } \frac{1}{3} 
\end{cases} \]

Separate: \[ R = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \]
Two Goods: Example 3

\[ X, Y \sim \begin{cases} 
0 & \text{w/probability } 1/3 \\
1 & \text{w/probability } 1/3 \quad \text{(I.I.D.)} \\
2 & \text{w/probability } 1/3 
\end{cases} \]

- **Separate:** \( R = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \)
- **Bundled:**
Two Goods: Example 3

\[ X, Y \sim \begin{cases} 
0 & \text{w/probability } 1/3 \\
1 & \text{w/probability } 1/3 \quad \text{(I.I.D.)} \\
2 & \text{w/probability } 1/3 
\end{cases} \]

- **Separate:** \( R = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \)
- **Bundled:** \( R = 2 \cdot \frac{6}{9} = \frac{4}{3} \)
Two Goods: Example 3

\[ X, Y \sim \begin{cases} 
0 & \text{w/probability } 1/3 \\
1 & \text{w/probability } 1/3 \\
2 & \text{w/probability } 1/3 
\end{cases} \quad \text{(I.I.D.)} \]

- Separate: \( R = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \)
- Bundled: \( R = 2 \cdot \frac{6}{9} = \frac{4}{3} \)

\[ b(x, y) = \max(0, x - 2, y - 2, x + y - 3) \]
Two Goods: Example 3

\[ X, Y \sim \begin{cases} 
0 & \text{w/probability } \frac{1}{3} \\
1 & \text{w/probability } \frac{1}{3} \quad \text{(I.I.D.)} \\
2 & \text{w/probability } \frac{1}{3} 
\end{cases} \]

- **Separate:** \( R = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \)
- **Bundled:** \( R = 2 \cdot \frac{6}{9} = \frac{4}{3} \)

\[ b(x, y) = \max(0, x - 2, y - 2, x + y - 3) \]
\[ R = \frac{13}{9} \]
Two Goods: Example 3

\[ X, Y \sim \begin{cases} 
0 & \text{w/probability } 1/3 \\
1 & \text{w/probability } 1/3 \\
2 & \text{w/probability } 1/3 
\end{cases} \quad \text{(I.I.D.)} \]

- **Separate**: \[ R = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \]
- **Bundled**: \[ R = 2 \cdot \frac{6}{9} = \frac{4}{3} \]

\[ b(x, y) = \max(0, x - 2, y - 2, x + y - 3) \]

\[ R = \frac{13}{9} \]
Two Goods: Example 3

\[ X, Y \sim \begin{cases} 
0 & \text{w/probability } \frac{1}{3} \\
1 & \text{w/probability } \frac{1}{3} \\
2 & \text{w/probability } \frac{1}{3}
\end{cases} \quad \text{(I.I.D.)} \]

- **Separate:** \[ R = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \]
- **Bundled:** \[ R = 2 \cdot \frac{6}{9} = \frac{4}{3} \]

\[ b(x, y) = \max(0, x - 2, y - 2, x + y - 3) \]
\[ R = \frac{13}{9} = \text{Rev}(X, Y) \]

**THE UNIQUE OPTIMAL MECHANISM**
Two Goods: Example 3

\[
X, Y \sim \begin{cases} 
0 & \text{w/probability } 1/3 \\
1 & \text{w/probability } 1/3 \\
2 & \text{w/probability } 1/3 
\end{cases} \quad \text{(I.I.D.)}
\]

Separate: \( R = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \)

Bundled: \( R = 2 \cdot \frac{6}{9} = \frac{4}{3} \)

\( b(x, y) = \max(0, x - 2, y - 2, x + y - 3) \)

\[
R = \frac{13}{9} = \text{Rev}(X, Y)
\]

THE UNIQUE OPTIMAL MECHANISM

PRICE FOR EACH GOOD AND FOR BUNDLE
Two Goods: Example 4
Two Goods: Example 4

\[(X, Y) \sim \begin{cases} 
(1, 0) & \text{w/probability } 1/3 \\
(0, 2) & \text{w/probability } 1/3 \\
(3, 3) & \text{w/probability } 1/3 
\end{cases}\]
Two Goods: Example 4

\[(X, Y) \sim \begin{cases} 
(1, 0) \quad \text{w/probability } 1/3 \\
(0, 2) \quad \text{w/probability } 1/3 \\
(3, 3) \quad \text{w/probability } 1/3 
\end{cases}\]

\[b(x, y) = \max(0, \frac{1}{2}x - \frac{1}{2}, y - 2, x + y - 5)\]
Two Goods: Example 4

\[(X, Y) \sim \begin{cases} 
(1, 0) & \text{w/probability } 1/3 \\
(0, 2) & \text{w/probability } 1/3 \\
(3, 3) & \text{w/probability } 1/3 
\end{cases} \]

\[b(x, y) = \max(0, \frac{1}{2}x - \frac{1}{2}, y - 2, x + y - 5)\]

\[R = 2.5\]
Two Goods: Example 4

\[(X, Y) \sim \begin{cases} 
(1, 0) & \text{w/probability } 1/3 \\
(0, 2) & \text{w/probability } 1/3 \\
(3, 3) & \text{w/probability } 1/3 
\end{cases} \]

\[b(x, y) = \max(0, \frac{1}{2}x - \frac{1}{2}, y - 2, x + y - 5)\]

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Two Goods: Example 4

\[(X, Y) \sim \begin{cases} 
(1, 0) & \text{w/probability } 1/3 \\
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\end{cases}\]

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THE UNIQUE OPTIMAL MECHANISM
Two Goods: Example 4

\[(X, Y) \sim \begin{cases} 
(1, 0) & \text{w/probability } 1/3 \\
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\end{cases}\]

\[b(x, y) = \max(0, \frac{1}{2}x - \frac{1}{2}, y - 2, x + y - 5)\]

\[R = 2.5 = \text{REV}(X, Y)\]

THE UNIQUE OPTIMAL MECHANISM

\[b_1(x, y) = \max(0, x - 1, y - 2, x + y - 4)\]
Two Goods: Example 4

\((X, Y) \sim \begin{cases} 
(1, 0) & \text{w/probability } 1/3 \\
(0, 2) & \text{w/probability } 1/3 \\
(3, 3) & \text{w/probability } 1/3 
\end{cases}\)

\[b(x, y) = \max(0, \frac{1}{2}x - \frac{1}{2}, y - 2, x + y - 5)\]

\[R = 2.5 = \text{REV}(X, Y)\]

**The Unique Optimal Mechanism**

\[b_1(x, y) = \max(0, x - 1, y - 2, x + y - 4)\]

\[b_0(x, y) = \max(0, y - 2, x + y - 5)\]
Two Goods: Example 4

\[(X, Y) \sim \begin{cases} (1, 0) \text{ w/probability } 1/3 \\ (0, 2) \text{ w/probability } 1/3 \\ (3, 3) \text{ w/probability } 1/3 \end{cases}\]

\[b(x, y) = \max(0, \frac{1}{2}x - \frac{1}{2}, y - 2, x + y - 5)\]

\[R = 2.5 = \text{REV}(X, Y)\]

**THE UNIQUE OPTIMAL MECHANISM**

\[b_1(x, y) = \max(0, x - 1, y - 2, x + y - 4)\]

\[R = 2.33...\]

\[b_0(x, y) = \max(0, y - 2, x + y - 5)\]

\[R = 2.33...\]
Two Goods: Example 4

\[(X, Y) \sim \begin{cases} 
(1, 0) & \text{w/probability } 1/3 \\
(0, 2) & \text{w/probability } 1/3 \\
(3, 3) & \text{w/probability } 1/3 
\end{cases}\]

\[b(x, y) = \max(0, \frac{1}{2}x - \frac{1}{2}, y - 2, x + y - 5)\]

\[R = 2.5 = \text{REV}(X, Y)\]

THE UNIQUE OPTIMAL MECHANISM

PRICE FOR LOTTERIES ON GOODS
Multiple Goods
Multiple Goods

Revenue maximizing mechanisms:
Revenue maximizing mechanisms:

1. post a price for each good separately
Multiple Goods

Revenue maximizing mechanisms:
1. post a price for each good separately
2. post a price for the bundle
Multiple Goods

Revenue maximizing mechanisms:
1. post a price for each good separately
2. post a price for the bundle
3. post prices for each good separately and for the bundle
Revenue maximizing mechanisms:
1. post a price for each good separately
2. post a price for the bundle
3. post prices for each good separately and for the bundle
4. post prices for various lotteries (or: fractional quantities)
Multiple Goods

Revenue maximizing mechanisms:
1. post a price for each good separately
2. post a price for the bundle
3. post prices for each good separately and for the bundle
4. post prices for various lotteries (or: fractional quantities)

Multiple Goods, I.I.D. Uniform
Multiple Goods, I.I.D. Uniform

\[ X_1, X_2, \ldots, X_k \sim \text{Uniform } [0, 1], \text{ i.i.d.} \]
Multiple Goods, I.I.D. Uniform

\[ X_1, X_2, \ldots, X_k \sim \text{Uniform } [0, 1], \text{ i.i.d.} \]

- \( k = 1: b(x) = \max(0, x_1 - \frac{1}{2}) \)
Multiple Goods, I.I.D. Uniform

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- \( k = 1: b(x) = \max(0, x_1 - \frac{1}{2}) \)

- \( k = 2: \)
  \[ b(x) = \max(0, x_i - \frac{2}{3}, x_1 + x_2 - \frac{4-\sqrt{2}}{3}) \]
Multiple Goods, I.I.D. Uniform

\(X_1, X_2, \ldots, X_k \sim \text{Uniform } [0, 1], \text{ i.i.d.}\)

- \(k = 1: b(x) = \max(0, x_1 - \frac{1}{2})\)
- \(k = 2:\)
  \[b(x) = \max(0, x_i - \frac{2}{3}, x_1 + x_2 - \frac{4 - \sqrt{2}}{3})\]
- \(k = 3: b(x) = \max(0, x_i - \frac{3}{4}, x_i + x_j - \frac{6 - \sqrt{2}}{4}, x_1 + x_2 + x_3 - s)\)
Multiple Goods, I.I.D. Uniform

\( X_1, X_2, \ldots, X_k \sim \text{Uniform} [0, 1], \text{ i.i.d.} \)

- \( k = 1 \): \( b(x) = \max(0, x_1 - \frac{1}{2}) \)
- \( k = 2 \):
  \[ b(x) = \max(0, x_i - \frac{2}{3}, x_1 + x_2 - \frac{4 - \sqrt{2}}{3}) \]
- \( k = 3 \): \( b(x) = \max(0, x_i - \frac{3}{4}, x_i + x_j - \frac{6 - \sqrt{2}}{4}, x_1 + x_2 + x_3 - s) \)

where \( s = \frac{9}{4} - \frac{\sqrt{6}}{4} \cos\left(\frac{1}{3} \arctan\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\right) \)

\[ -\frac{3\sqrt{2}}{4} \sin\left(\frac{1}{3} \arctan\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\right) \]
Multiple Goods, I.I.D. Uniform

\(X_1, X_2, \ldots, X_k \sim \text{Uniform } [0, 1], \text{ i.i.d.}\)

- \(k = 1:\ b(x) = \max(0, x_1 - \frac{1}{2})\)
- \(k = 2:\ b(x) = \max(0, x_i - \frac{2}{3}, x_1 + x_2 - \frac{4-\sqrt{2}}{3})\)
- \(k = 3:\ b(x) = \max(0, x_i - \frac{3}{4}, x_i + x_j - \frac{6-\sqrt{2}}{4}, x_1 + x_2 + x_3 - s)\)

where \(s \approx 1.2257\ldots = \text{solution of 3rd degree equation with coefficients in } \mathbb{Q}[\sqrt{2}]\)
Multiple Goods, I.I.D. Uniform

\[ X_1, X_2, \ldots, X_k \sim \text{Uniform } [0, 1], \text{ i.i.d.} \]

- \( k = 1: \ b(x) = \max(0, \ x_1 - \frac{1}{2}) \)

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- \( k = 3: \ b(x) = \max(0, \ x_i - \frac{3}{4}, \ x_i + x_j - \frac{6-\sqrt{2}}{4}, \ x_1 + x_2 + x_3 - s) \)

\ldots
Multiple Goods, I.I.D. Uniform

\(X_1, X_2, ..., X_k \sim \text{Uniform } [0, 1], \text{ i.i.d.}\)

- \(k = 1: b(x) = \max(0, x_1 - \frac{1}{2})\)
- \(k = 2:\)
  \(b(x) = \max(0, x_i - \frac{2}{3}, x_1 + x_2 - \frac{4-\sqrt{2}}{3})\)
- \(k = 3: b(x) = \max(0, x_i - \frac{3}{4}, x_i + x_j - \frac{6-\sqrt{2}}{4}, x_1 + x_2 + x_3 - s)\)
  . . .

Manelli & Vincent 2006, Hart & Reny 2010
Monotonicity

If valuations of **BUYER** increase
Monotonicity

If valuations of **BUYER** increase then the maximal revenue of **SELLER** increases (weakly)
Monotonicity

If valuations of **BUYER** increase then the maximal revenue of **SELLER** increases (weakly)

**Proof for** $k = 1$:
Monotonicity

If valuations of **BUYER** increase then the maximal revenue of **SELLER** increases (weakly)

**Proof for** $k = 1$:
- Every **IC** mechanism has **monotonic** $s$
Monotonicity

If valuations of **BUYER** increase then the maximal revenue of **SELLER** increases (weakly)

**Proof for** $k = 1$:

- Every **IC** mechanism has **monotonic** $s$
- $\Rightarrow$ Revenue of every **IC** mechanism is **monotonic** w.r.t. to **BUYER** valuations
Monotonicity

If valuations of **BUYER** increase then the maximal revenue of **SELLER** increases (weakly)

**Proof for** $k = 1$:
- Every **IC** mechanism has **monotonic**
- $\Rightarrow$ Revenue of every **IC** mechanism is **monotonic** w.r.t. to **BUYER** valuations
- $\Rightarrow$ Maximal revenue is **monotonic** w.r.t. **BUYER** valuations
Monotonicity

If valuations of **BUYER** *increase* then the maximal revenue of **SELLER** *increases* (weakly)

**Proof for** $k = 1$:

- Every **IC** mechanism has **monotonic**
- $\Rightarrow$ Revenue of every **IC** mechanism is **monotonic** w.r.t. **BUYER** valuations
- $\Rightarrow$ Maximal revenue is **monotonic** w.r.t. **BUYER** valuations

**Proof for** $k > 1$ ?
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]
$$b(x, y) = \max(0, x - 10, y - 20, x + y - 40)$$
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]

\( x \) increases
\( y \) increases
\( s \) DECREASES!

\((12, 24) : y - 20\)
\((18, 26) : x - 10\)
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]

There exist distributions \( F \) for which this \( b \) MAXIMIZES REVENUE
Non-Monotonicity: Example

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]

There exist distributions \( F \) for which this \( b \) MAXIMIZES REVENUE (moreover: unique maximizer; robust)
Non-Monotonicity

\[ b(x, y) = \max(0, x - 10, y - 20, x + y - 40) \]

- There exist distributions \( F \) for which this \( b \) \textit{MAXIMIZES REVENUE} (moreover: unique maximizer; robust)

- \textbf{NON-MONOTONICITY} occurs also for \textit{I.I.D.}
Summary: Multiple Goods
Summary: Multiple Goods

Maximizing revenue with multiple goods:
Maximizing revenue with multiple goods:

- many of the results for ONE GOOD are FALSE for MULTIPLE GOODS
Maximizing revenue with multiple goods:

- many of the results for ONE GOOD are FALSE for MULTIPLE GOODS
- is an extremely complex problem (even for simple distributions)
Maximizing revenue with multiple goods:
- many of the results for ONE GOOD are FALSE for MULTIPLE GOODS
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Summary: Multiple Goods

Maximizing revenue with multiple goods:

- many of the results for **ONE GOOD** are **FALSE** for **MULTIPLE GOODS**
- is an extremely complex problem (even for simple distributions)
- "what we have learned from one good is too good to be true for two goods"
- ?
Maximizing revenue with multiple goods:

- many of the results for **ONE GOOD** are **FALSE** for **MULTIPLE GOODS**
- is an extremely complex problem (even for simple distributions)
- "what we have learned from one good is too good to be true for two goods"
- ?
- **APPROXIMATION** using **SIMPLE** mechanisms ?
Two Goods, Independent
Two Goods, Independent

**MAXIMAL REVENUE** of selling **SEPARATELY**:

\[
S\text{Rev}(F_1 \times F_2) = R\text{ev}(F_1) + R\text{ev}(F_2)
\]
Two Goods, Independent

MAXIMAL REVENUE of selling SEPARATELY:

\[ S\text{Rev}(F_1 \times F_2) = \text{Rev}(F_1) + \text{Rev}(F_2) \]

Theorem. For any two independent goods:

\[ S\text{Rev}(F_1 \times F_2) \geq \frac{1}{2} \text{Rev}(F_1 \times F_2) \]
Two Goods, Independent

**MAXIMAL REVENUE** of selling **SEPARATELY**:

\[ \text{SRev}(F_1 \times F_2) = \text{Rev}(F_1) + \text{Rev}(F_2) \]

---

**Theorem.** For any two independent goods:

\[ \text{SRev}(F_1 \times F_2) \geq \frac{1}{2} \text{Rev}(F_1 \times F_2) \]

**Theorem.** For any two i.i.d. goods:

\[ \text{SRev}(F \times F) \geq \frac{e}{e+1} \text{Rev}(F \times F) \]

\[ \frac{e}{e+1} \approx 73\% \]
Two Goods, I.I.D.

Theorem. For any two i.i.d. goods:

$$\text{SREV}(F \times F) \geq \frac{e}{e+1} \text{REV}(F \times F)$$

$$\frac{e}{e+1} \approx 73\%$$
Two Goods, I.I.D.

Theorem. For any two i.i.d. goods:

\[ \text{SRev}(F \times F) \geq \frac{e}{e+1} \text{Rev}(F \times F) \]

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Theorem. For any two i.i.d. goods:

\[ \text{SREV}(F \times F) \geq \frac{e}{e+1} \text{REV}(F \times F) \]

\[ \frac{e}{e+1} \approx 73\% \]

\[ \text{SREV}(F \times F) = 2 \text{REV}(F) = 2 p^* \cdot (1 - F(p^*)) \]
Two Goods, I.I.D.

**Theorem.** For any two i.i.d. goods:

\[
\text{SR} \text{EV}(F \times F) \geq \frac{e}{e+1} \text{REV}(F \times F)
\]

\[
\frac{e}{e+1} \approx 73\%
\]

\[
\text{SR} \text{EV}(F \times F) = 2 \text{REV}(F) = 2 p^* \cdot (1 - F(p^*))
\]

Posting the optimal one-good price per unit guarantees at least 73% of the optimal revenue.
Consider

A class of IC & IR mechanisms $M$
Consider

- A class of IC & IR mechanisms $M$
- A family of distributions $F$
Consider

- A class of IC & IR mechanisms $M$
- A family of distributions $F$

**Guaranteed Fraction of Optimal Revenue**
Consider

- A class of IC & IR mechanisms $M$
- A family of distributions $F$

**GUARANTEED FRACTION OF OPTIMAL REVENUE**
Consider

- A class of IC & IR mechanisms $\mathcal{M}$
- A family of distributions $\mathcal{F}$

**Guaranteed Fraction of Optimal Revenue**

maximal fraction $0 \leq \alpha \leq 1$ such that
for every distribution $F$ in $\mathcal{F}$
there is a mechanism $\mathcal{M}$ in $\mathcal{M}$ with

$$R(\mathcal{M}, F) \geq \alpha \cdot \text{Rev}(F)$$
GFOR: Two Goods, Independent
GFOR: Two Goods, Independent

**SEPARATE** selling of **INDEPENDENT** goods:
GFOR: Two Goods, Independent

Separate selling of independent goods:

\[ \text{GFOR} \geq 50\% \]
GFOR: Two Goods, Independent

- **SEPARATE** selling of **INDEPENDENT** goods:
  \[ \text{GFOR} \geq 50\% \]

- **SEPARATE** selling of **I.I.D.** goods:
GFOR: Two Goods, Independent

- **SEPARATE** selling of **INDEPENDENT** goods:
  \[ \text{GFOR} \geq 50\% \]

- **SEPARATE** selling of **I.I.D.** goods:
  \[ \text{GFOR} \geq 73\% \]
GFOR: Two Goods, Independent

1 BUYER, 2 GOODS

- SEPARATE selling of INDEPENDENT goods:
  \[ \text{GFOR} \geq 50\% \]

- SEPARATE selling of I.I.D. goods:
  \[ \text{GFOR} \geq 73\% \]
GFOR: Two Goods, Independent

1 BUYER, 2 GOODS

- SEPARATE selling of INDEPENDENT goods:
  \[ \text{GFOR} \geq 50\% \]

- SEPARATE selling of I.I.D. goods:
  \[ \text{GFOR} \geq 73\% \]

n BUYERS, 2 GOODS
GFOR: Two Goods, Independent

1 BUYER, 2 GOODS

- SEPARATE selling of INDEPENDENT goods:
  \[ GFOR \geq 50\% \]

- SEPARATE selling of I.I.D. goods:
  \[ GFOR \geq 73\% \]

n BUYERS, 2 GOODS

- SEPARATE selling of INDEPENDENT goods:
GFOR: Two Goods, Independent

1 BUYER, 2 GOODS

- SEPARATE selling of INDEPENDENT goods:
  \[ \text{GFOR} \geq 50\% \]

- SEPARATE selling of I.I.D. goods:
  \[ \text{GFOR} \geq 73\% \]

\( n \) BUYERS, 2 GOODS

- SEPARATE selling of INDEPENDENT goods:
  \[ \text{GFOR} \geq 50\% \]
2 GOODS, ARBITRARY DEPENDENCE
2 GOODS, ARBITRARY DEPENDENCE

- SEPARATE selling:
2 GOODS, ARBITRARY DEPENDENCE

- SEPARATE selling: \( \text{GFOR} = 0 \).
2 GOODS, ARBITRARY DEPENDENCE

- SEPARATE selling: \( \text{GFOR} = 0 \).

For every \( \varepsilon > 0 \) there is a distribution \( F \) on \([0, 1]^2\) such that: \( \text{SR}_{\text{Rev}}(F) < \varepsilon \text{ Rev}(F) \).
2 GOODS, ARBITRARY DEPENDENCE

- **SEPARATE** selling: \( \text{GFOR} = 0 \).

- **BUNDLED** selling:
2 GOODS, ARBITRARY DEPENDENCE

- **SEPARATE** selling:  \( \text{GFOR} = 0. \)
- **BUNDLED** selling:  \( \text{GFOR} = 0. \)
for every $\varepsilon > 0$ there is a distribution $F$ on $[0, 1]^2$ such that $\text{BRev}(F) < \varepsilon \text{ Rev}(F)$
GFOR: General

2 GOODS, ARBITRARY DEPENDENCE

- SEPARATE selling: \( \text{GFOR} = 0 \).
- BUNDLED selling: \( \text{GFOR} = 0 \).
- DETERMINISTIC mechanisms:
GFOR: General

2 GOODS, ARBITRARY DEPENDENCE

- **SEPARATE** selling: \( \text{GFOR} = 0 \).
- **BUNDLED** selling: \( \text{GFOR} = 0 \).
- **DETERMINISTIC** mechanisms: \( \text{GFOR} = 0 \).
**GFOR: General**

### 2 GOODS, ARBITRARY DEPENDENCE

- **SEPARATE** selling: $\text{GFOR} = 0$.
- **BUNDLED** selling: $\text{GFOR} = 0$.
- **DETERMINISTIC** mechanisms: $\text{GFOR} = 0$.

For every $\varepsilon > 0$ there is a distribution $F$ on $[0, 1]^2$ such that $\text{DRev}(F) < \varepsilon \text{ Rev}(F)$. 
[\[m\]]-REV(F) = maximal REVENUE from mechanisms with AT MOST m OUTCOMES
$[m] - \text{Rev}(F) = \text{maximal revenue from mechanisms with at most } m \text{ outcomes}$

- $[m] - \text{Rev}$ for fixed $m$: $\text{GFOR} = 0$
$[m]$-\textbf{Rev}(F) = \text{maximal Revenue from mechanisms with at most } m \text{ outcomes}$

- $[m]$-\textbf{Rev} for fixed $m$: $\text{GFOR} = 0$
- $[m]$-\textbf{Rev} increases with $m$ (polynomially)
$[m]\text{-Rev}(F) = \text{maximal REVENUE from mechanisms with AT MOST } m \text{ OUTCOMES}$

- $[m]\text{-Rev}$ for fixed $m$: $\text{GFOR} = 0$
- $[m]\text{-Rev}$ increases with $m$ (polynomially)
- DETERMINISTIC-Rev $\sim [2^k]\text{-Rev}$
\([m]-\text{Rev}(F) = \text{maximal revenue}\) from mechanisms with \textbf{AT MOST} \(m\) \textbf{OUTCOMES}

- \([m]-\text{Rev}\) for fixed \(m\): \(\text{GFOR} = 0\)
- \([m]-\text{Rev}\) increases with \(m\) (polynomially)
- \textbf{Deterministic-Rev} \(\sim [2^k]-\text{Rev}\)

\textbf{Menu Size} = \text{measure of the COMPLEXITY of mechanisms}
Summary: Multiple Goods
Maximizing revenue with multiple goods:
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- many of the results for **ONE GOOD** are **FALSE** for **MULTIPLE GOODS**
Summary: Multiple Goods

Maximizing revenue with multiple goods:
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Maximizing revenue with multiple goods:

- many of the results for **ONE GOOD** are **FALSE** for **MULTIPLE GOODS**
- is an extremely complex problem (even for simple distributions)
- "**what we have learned from one good is too good to be true for two goods**"
- **SIMPLE** mechanisms **MAY** yield **UNIFORM** APPROXIMATION
Two(!) Good To Be True
"Are you trying to auction your Brussels sprouts again?"