

# Global Analysis and Automorphic Forms

## In honor of the 60th birthday of Werner Müller

Hebrew University of Jerusalem, Israel

March 14–20, 2010

Sponsored by

- The Edmund Landau Minerva Center for Research in Mathematical Analysis and Related Areas
- SFB 701 (Bielefeld): Spektrale Strukturen und Topologische Methoden in der Mathematik
- The Hebrew University of Jerusalem

## Foreign participants

Jean-Michel Bismut (Orsay)  
Jochen Brüning (Humboldt)  
Ulrich Bunke (Regensburg)  
Leonardo Cano (Bonn)  
Jeff Cheeger (New York University)  
Xianzhe Dai (Santa Barbara)  
Jürgen Eichhorn (Greifswald)  
Tobias Finis (Düsseldorf)  
Fritz Grunewald (Düsseldorf)  
Lizhen Ji (Michigan)  
Dihua Jiang (Minnesota)  
Bernhard Krötz (Hannover)  
Kai Köhler (Düsseldorf)  
Herbert Kurke (Humboldt)  
Sweitse van Leeuwen (Hannover)  
Matthias Lesch (Bonn)  
Christoph Lienau (Hannover)  
Jasmin Matz (Düsseldorf)  
Richard Melrose (MIT)  
Tomonori Moriyama (Osaka)  
Jörn Müller (Humboldt)

Werner Müller (Bonn)  
Martin Olbrich (Luxemburg)  
Michael Ontiveros (MPI Bonn)  
Aprameyan Parthasarathy (Göttingen)  
Yiannis Petridis (London)  
Jonathan Pfaff (Bonn)  
Jens Putzka (Bonn)  
Pablo Ramacher (Marburg)  
Jürgen Rohlf's (Eichstätt)  
Robert Schrader (Berlin)  
Joachim Schwermer (Vienna)  
Freydoon Shahidi (Purdue)  
Birgit Speh (Cornell)  
Robert Stanton (Columbus)  
Alexander Strohmaier (Loughborough)  
Octavio Taboada (Göttingen)  
Masao Tsuzuki (Tokyo)  
Boris Vertman (Stanford)  
Satoshi Wakatsuki (Kanazawa)  
Kartin Wendland (Augsburg)

# PROGRAM

Sunday 14/3	Monday 15/3	Tuesday 16/3	Wednesday 17/3	Thursday 18/3	Friday 19/3	Saturday 20/3
8:00-9:00 <i>Breakfast</i>	8:00-9:00 <i>Breakfast</i>	8:00-9:00 <i>Breakfast</i>	8:00-9:00 <i>Breakfast</i>	8:00-9:00 <i>Breakfast</i>	8:00-9:00 <i>Breakfast</i>	7:30-8:30 <i>Breakfast</i>
	9:30-10:30 Petridis	9:30-10:30 Finis	9:15-10:15 Brüning	9:30-10:30 Kazhdan	9:15-10:15 Krötz	8:30-17:00 <sup>1</sup>
	11:00-12:00 Lesch	11:00-12:00 Bismut	10:30-11:30 Ramacher	11:00-12:00 Shahidi	10:30-11:30 Olbrich	<i>Excursion to Masada and the Dead Sea</i>
			11:45-12:45 Schrader		11:45-12:45 Wendland	
12:30-14:00 <i>Lunch</i>	12:30-14:00 <i>Lunch</i>	12:30-14:00 <i>Lunch</i>	13:00-14:00 <i>Lunch</i>	12:30-14:00 <i>Lunch</i>	13:00-14:00 <i>Lunch</i>	
14:30-15:30 Bunke	14:30-15:30 Stanton	14:30-15:30 Grunewald	14:00-19:00 <sup>2</sup>	14:30-15:30 Melrose <sup>3</sup>		
			<i>Excursion</i>	<i>coffee break</i>		
16:00-17:00 Eichhorn	16:00-17:00 Ji	16:00-17:00 Strohmaier	<i>to the old</i>	16:00-17:00 Cheeger <sup>4</sup>		
<i>coffee break</i>	<i>coffee break</i>	<i>coffee break</i>				
17:30-18:30 Jiang	17:30-18:30 Schwermer	17:30-18:30 Dai	<i>city</i>	17:15-18:30 Speh <sup>5</sup>		
19:00 <i>Reception</i> <sup>6</sup>			19:00 <i>Banquet</i> <sup>7</sup>			

All lectures take place at the main hall (Ulam 2) of the Manchester building (Mathematics), on Givat Ram campus.

Breakfast is served at the Park Hotel.

Lunches are served at Bet Belgia dining hall.

<sup>1</sup>bus departs at 8:30 from the Park Hotel

<sup>2</sup>bus departs at 14:00 from Bet Belgia

<sup>3</sup>joint with Analysis seminar

<sup>4</sup>joint with Department Colloquium

<sup>5</sup>joint with "Basic Notions" seminar

<sup>6</sup>at Bet Belgia library hall

<sup>7</sup>Darna restaurant, 3 Horkanos St., Jerusalem

## Abstracts

- **Jean-Michel Bismut** *Hypoelliptic Laplacian and orbital integrals*

The hypoelliptic Laplacian is a natural family of second order operators acting on the total space of the (co)tangent bundle of a smooth compact manifold, which interpolates between the classical Hodge Laplacian (in de Rham or Dolbeault theory) and the geodesic flow. It is essentially a weighted sum of the harmonic oscillator along the fiber and of the vector field generating the geodesic flow. This hypoelliptic deformation comes itself from a deformation of the associated Hodge theory, and of the corresponding Dirac operator. The analytic properties of the hypoelliptic Laplacian have been established by Lebeau and ourselves. If  $G$  is a reductive group with Lie algebra  $\mathfrak{g}$ , we have applied this method to the explicit evaluation of semisimple orbital integrals. If  $X = G/K$  is the associated symmetric space, the hypoelliptic deformation of the Casimir acts on  $X \times \mathfrak{g}$ . The orbital integrals are shown to be independent of the deformation parameter. Localization on closed geodesics leads to an explicit formula for the orbital integrals. In this lecture, I will review the above constructions and results.

- **Jochen Brüning** *A gluing formula for the analytic torsion*

I will report on recent work with Xiaonan Ma. We consider the analytic torsion on a compact Riemannian manifold with boundary for differential forms with values in a hermitian flat vector bundle and subject to absolute or relative boundary conditions. We derive the gluing formula for this situation without assuming that the metric on the flat vector bundle is flat nor that the Riemannian metric has product structure near the boundary.

- **Ulrich Bunke** *Secondary invariants for the Witten genus*

We construct secondary and tertiary invariants from complex elliptic genera and the Witten genus. Using index theory for manifolds with corners these invariants can be expressed in terms of eta-invariants of Dirac operators. We will calculate the invariants and discuss their topological significance.

- **Jeff Cheeger** (colloquium talk) *Lipschitz maps from metric spaces with many rectifiable curves*

We will give an overview of work during roughly the past 10 years on the subject in the title. Some of this is joint with B. Kleiner and with Kleiner and A. Naor. We hope to cover differentiability of real valued Lipschitz functions, Banach space targets with the Radon-Nikodym-Property, the Carnot-Carathéodory/Heisenberg example, quantitative bi-Lipschitz nonembedding for Heisenberg in  $L_1$  and applications to the worst case performance of the Goemans-Linial algorithm for the sparsest cut problem with general demands.

- **Xianzhe Dai** *The analytic and Reidemeister torsions for manifolds with conical singularity*

The Ray-Singer Conjecture, which is now the celebrated Cheeger-Müller Theorem, says that the analytic torsion equals the Reidemeister torsion on closed manifolds. For manifolds with conical singularity, the analytic torsion can be defined using Cheeger's theory on conical singularity, while the Reidemeister torsion can be defined using the intersection cohomology of Goresky-MacPherson. It is an intriguing question whether and how Ray-Singer Conjecture generalizes to this setting. We will discuss some recent work in this direction.

- **Jürgen Eichhorn** *Zeta function techniques for open manifolds*

For non-negative elliptic differential operators on closed manifolds, we have a well established index theory (in the case of a grading) and the notion of analytic torsion. On open manifolds, this becomes wrong since such operators must not have a finite dimensional kernel and cokernel and the spectrum must not be purely discrete. But there is a possibility to establish a similar theory for pairs of operators  $D, D'$  where  $D'$  is an appropriate perturbation of  $D$ . We consider pairs  $D^2, D'^2$ , where  $D, D'$  are generalized Dirac operators (e.g. graded Laplace operators) and  $D'$  is an appropriate perturbation of  $D$ . Doing this, there arise two canonical questions: 1) What are the appropriate perturbations and 2) is  $e^{-tD^2} - e^{-t\tilde{D}'^2}$  a trace class operator, where  $\tilde{D}'$  is a certain transformation of  $D'$ ? We answer these two questions and establish in fact a very general relative index and analytic torsion theory.

- **Tobias Finis** *Arthur's trace formula and spectral theory*

The talk will give an introduction to Arthur's trace formula with a focus on analytic problems and applications to spectral theory. It will discuss Arthur's basic regularization procedure as well as recent results on the absolute convergence of the spectral side and Weyl's law for cusp forms.

- **Fritz Grunewald** *Cohomology of lattices in  $SL(2, \mathbb{C})$*

Formulas for the dimensions of spaces of classical modular forms are well known. There are many ways to derive them. The methods range from linear algebra or the theorem of Riemann-Roch to the Selberg trace formula. All these methods are much less powerful when the field of rational numbers is replaced by an imaginary quadratic number field. In my talk I will describe the state of our knowledge and also some conjectures in this case.

- **Lizhen Ji** *A tale of two spaces: locally symmetric spaces and moduli space of curves*

This talk will explain some recent results showing similarities, differences and interactions between two important classes of spaces: locally symmetric spaces that are quotients of symmetric spaces by arithmetic groups, and the moduli spaces of Riemann surfaces, which are quotients of the Teichmüller spaces by mapping class groups.

- **Dihua Jiang** *Automorphic descents and certain Arthur packets*

In this talk, I will report the recent progress of my joint work with David Ginzburg and David Soudry on the constructions of automorphic transfers, which is generalization of the automorphic descents originally introduced by Ginzburg-Rallis-Soudry in late 90's. These constructions yield relations among global Arthur packets of certain. including the endoscopy transfers among the tempered Arthur packets.

- **David Kazhdan** *affine Gindikin-Karpelevich formula via Uhlenbeck spaces*

- **Bernhard Krötz** *Analytic representation theory*

We will introduce the notion of (tempered) analytic Lie group representations and discuss issues as factorization or analytic globalization of Harish-Chandra modules.

- **Matthias Lesch** *Connes-Chern character for manifolds with boundary*

We represent the Connes-Chern character of the Dirac operator associated to a  $b$ -metric on a manifold with boundary in terms of a retracted cocycle in relative cyclic cohomology, whose expression depends on a scaling/cut-off parameter. Blowing-up the metric one recovers the pair of characteristic currents that represent the corresponding de Rham relative homology class, while the blow-down yields a relative cocycle whose expression involves higher eta cochains and their  $b$ -analogues. The corresponding pairing formulae with relative  $K$ -theory classes capture information about the boundary, and thus have geometric implications. As a byproduct we obtain a generalization of the Atiyah-Patodi-Singer odd-index theorem, from trivialized flat bundles to any pair of  $K$ -equivalent vector bundles.

The talk is based on recent joint work with Henri Moscovici and Markus Pflaum.

- **Richard Melrose** *Equivariant resolution, cohomology and index*

I will describe the resolution to unique isotropy type of the smooth action of a compact Lie group as contained in joint work with Pierre Albin. The resulting 'resolution tower' leads to resolved and reduced models for equivariant cohomology, including the delocalized cohomology of Baum, Brylinski and MacPherson, and  $K$ -theory. Combining these constructions with a lifting map gives a families version of the pseudodifferential equivariant index theorem of Atiyah and Singer with corresponding representations of the Chern character.

- **Martin Olbrich** *A trace formula for quotients by convex cocompact groups*

I shall discuss variants of a trace formula for certain locally symmetric spaces of infinite volume. The goal is to find a spectral expression for a natural sum of hyperbolic orbital integrals. I am particularly interested in applications to Selberg type zeta functions. The talk is based on joint work with Ulrich Bunke.

- **Yiannis Petridis** *Dissolving cusp forms into resonances: Higher order Fermi's Golden Rules*

For a hyperbolic surface embedded eigenvalues are unstable and tend to become resonances. The sufficient dissolving condition was identified by Phillips-Sarnak and is elegantly expressed in Fermi's Golden Rule. We prove formulas for higher approximations and obtain necessary and sufficient conditions for dissolving a cusp form into a resonance. We relate the result to the special values of  $L$ -series involving a Rankin-Selberg convolution of the cusp form with higher order automorphic forms. This is joint work with Morten S. Risager.

- **Pablo Ramacher** *Singular reduced asymptotics and Weyl's law*

We discuss the spectrum of an invariant elliptic operator on a closed  $G$ -manifold  $M$ ,  $G$  being a compact, connected Lie group acting effectively and isometrically on  $M$ , and determine the asymptotic distribution of the eigenvalues along the isotypic components. While the leading term can be obtained via heat kernel methods, estimates for the remainder are not accessible with this approach. On the other hand, the derivation of remainder estimates within the framework of Fourier integral operators meets with serious difficulties when singular orbits are present. In this talk, we shall indicate how to circumvent these difficulties by using resolution of singularities, and prove that the equivariant spectral counting function satisfies Weyl's law, together with an estimate for the remainder.

- **Robert Schrader** *Finite propagation speed for solutions of the wave equation on metric graphs*

We discuss Laplace operators on metric graphs. A complete characterization of all self adjoint Laplacians is given in terms of the (local) boundary conditions they satisfy. Each such Laplacian defines a wave operator. For a certain subclass we show that solutions of corresponding the wave equation satisfy the finite propagation speed. The proof uses an essential modification of well known local energy estimates used for solutions of the wave equation on smooth manifolds. These results were obtained in part in collaboration with J. Potthoff and V. Kostrykin.

- **Joachim Schwermer** *Eisenstein series, cohomology of arithmetic groups and automorphic L-functions at half-integral arguments*

The automorphic cohomology of a reductive  $\mathbb{Q}$ -group  $G$ , defined in terms of the automorphic spectrum of  $G$ , captures essential analytic aspects of the arithmetic subgroups of  $G$  and their cohomology. The subspace spanned by all possible residues and principal values of derivatives of Eisenstein series, attached to cuspidal automorphic forms  $\pi$  on the Levi factor of proper parabolic  $\mathbb{Q}$ -subgroups of  $G$ , forms the Eisenstein cohomology which is a natural complement to the cuspidal cohomology. We discuss that non-trivial Eisenstein cohomology classes can only arise if the point of evaluation features a ‘half-integral’ property.

- **Freydoon Shahidi** *Arthur packets and the Ramanujan Conjecture for generic cusp forms*

In this talk we will show that under a part of Arthur’s  $A$ -packet conjecture, generic cuspidal automorphic representations of a quasisplit group over a number field are of Ramanujan type, i.e., are tempered at almost all places.

- **Birgit Speh** *Branching rules* (“basic notions” seminar)

Understanding a unitary representation  $\pi$  of a Lie groups  $G$  often involves understanding its restriction to suitable subgroups  $H$ . This is in physics referred to as breaking the symmetry, and often means exhibiting a nice model of the representation space of  $\pi$ . In this talk I will discuss some of the techniques and problems that arise. I will also explain some applications to the cohomology of arithmetic groups and locally symmetric spaces.

- **Robert Stanton** *Examples of the usefulness of symplectic methods*

We shall examine special symplectic modules for certain reductive groups. Using symplectic methods we will investigate in detail the orbits. The structure of the orbits will be shown to generate a symplectic tower of embedded dual pairs, sometimes referred to as the Rallis tower. Other results will be differential geometric ones obtained when the base field is specialized to  $\mathbb{R}$ , namely we obtain new examples of special pseudo-Kähler metrics. In a similar fashion, an alternate approach to certain Sato-Shintani zeta functions will be presented. This is joint with M. Slupinski.

- **Alex Strohmaier** *The scattering length of manifolds*

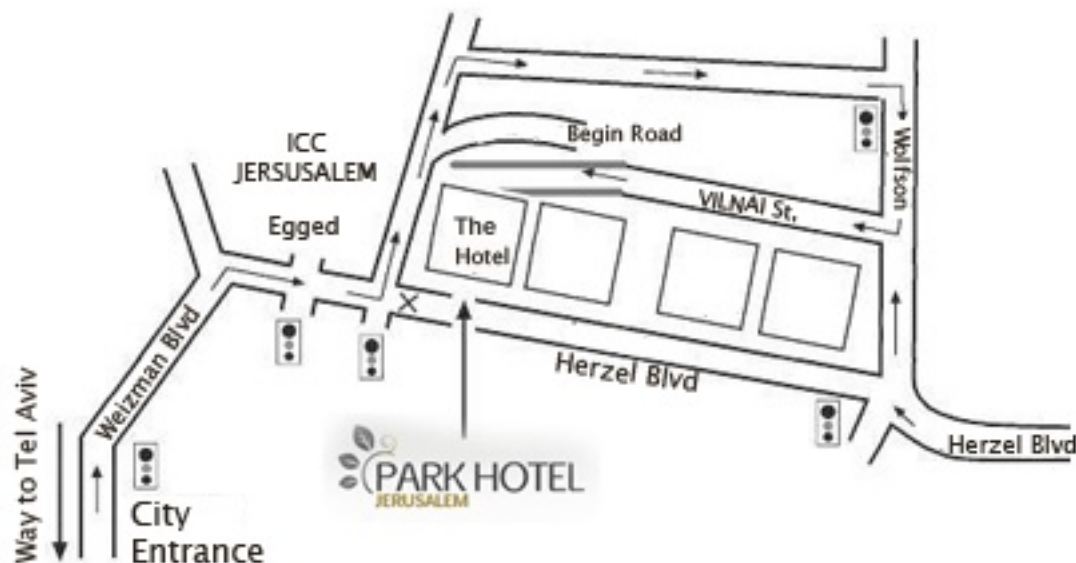
A quantum mechanical particle or a wave scattered in a manifold with cylindrical end exhibits a time delay due to the presence of the manifold. For waves with large wavelength this time delay is a geometric invariant of the manifold called the scattering length. We give some estimate of the scattering length in terms of other geometric data. This is a joint work with Werner Müller

- **Katrin Wendland** *On singularities and modularity*

A well established result in conformal field theory which is often assigned to Kiritsis states an ADE classification of so-called "unitary Virasoro minimal models", a family of particularly well behaved conformal field theories. This classification is, in many respects, parallel to the ADE classification of simple quotient singularities on complex surfaces due to Cartan, Prill, Brieskorn and Arnold. However, the characters of a conformal field theory and their modular transformation properties add an aspect of modularity, which is not present in the classical geometric picture. This idea can be generalized to other classes of singularities. The talk will attempt to describe this bridge between singularity theory and modularity, explaining the input from conformal field theory without assuming background knowledge on the latter topic from the audience.

## Practical information

Conference participants will stay at the Park Hotel, located at 2 Vilnay street close to the central bus station. See <http://www.park-hotel-jerusalem.com>



The simplest way to get to the hotel from Ben-Gurion airport is to take a taxi (around 200-250 NIS, depending on time of day). Without traffic it should take around 40 minutes. In principle it should be possible to pay with credit card, but double check with the driver. A cheaper, but less convenient way is to take a door-to-door shared taxi called “sherut” (operated by Neshet – around 50 NIS per person). This takes longer since you may have to wait until the van fills up and there may be several destinations before the Park Hotel. *In both cases, make sure the driver knows exactly where you are going (perhaps by showing him the attached map), since there are several hotels with similar names.* The Park Hotel is at the main entrance to the city, very close to the Calatrava Bridge of Strings.

*In both ways tipping is **not** customary.*

Unfortunately, there is no reasonable bus or train service from the airport.

Needless to say, **never leave your valuables (including passport) unattended.** The hotel provides free personal safe deposit boxes upon request.

To get to campus from the hotel simply follow Vilnay road past the hotels and cross the bridge above Begin road on Wolfson street. Then walk through Hebrew University Secondary School (on your right) to the main gate of the campus.

For campus map see <http://www.huji.ac.il/huji/maps/givatramCampus.htm> or <http://science.huji.ac.il/map/easymap/newmap.php>.

## Places to see in the area

There are several museums near the Hebrew University, within walking distance from the hotel. Unfortunately, the biggest attraction, the Israel Museum, is temporarily mostly closed for renovation. The adjacent *shrine of the book* is open though.

<http://www.english.imjnet.org.il/htmls/home.aspx>

The Bible Land Museum <http://www.blmj.org/en/index.php>

The Science Museum <http://www.mada.org.il/en/>

Another place of interest within a short taxi ride is Yad Vashem – the holocaust museum <http://www.yadvashem.org/>

Further away is the beautiful Ein Kerem neighborhood [http://en.wikipedia.org/wiki/Ein\\_Kerem](http://en.wikipedia.org/wiki/Ein_Kerem)

The rose garden near the Knesset (the Israeli parliament) is also worth visiting [http://en.wikipedia.org/wiki/Rose\\_Garden\\_\(Jerusalem\)](http://en.wikipedia.org/wiki/Rose_Garden_(Jerusalem))

Visiting the Mahane Yehuda market (the “shuk”) is also recommended.

## Places to eat

Most restaurants in Jerusalem are Kosher. They are subdivided into meat and dairy. In the first type there are no dairy products whatsoever. In the second type there are no meat products (but most fish is allowed).

The only place open for dinner on campus is the very reasonable dairy (+ fish) *Café Ha'ivrit* next to the gym and swimming pool.

The market area (around Agripas street) not too far from the hotel has many restaurants and food stalls of all kinds, mostly middle eastern.

A little closer is a nice fish restaurant (notwithstanding its less than ideal location) *ahavat hayam* located above (but insulated from) the gas station on Ben-Tzvi 11 (corner with Betzalel).

There is also a dairy restaurant in the botanical gardens (the far end of campus – entrance from the Bazak street).

Generally, Kosher restaurants are closed from Friday early afternoon until Saturday evening (around 7PM). The main exception is hotel restaurants, which are more expensive. The non-kosher restaurants are almost always open on Friday and Saturday. The largest cluster of non-kosher restaurants (as well as many kosher ones) is located on Shlomtzion street and the surrounding area (downtown Jerusalem). A partial list is available for instance on <http://www.arzaworld.com/israel-travel-guide/israel-restaurant-guide/jerusalem-restaurant-guide.aspx>

There is also a small grocery store on campus near the dorms.