

FLOWS ON HOMOGENEOUS SPACES - COURSE OUTLINE

ELON LINDENSTRAUSS

SYLLABUS

The space of lattices in $\mathbb{R}^n$.

- (0) Lattices and fundamental domains in locally compact groups.
- (1) identification of space of lattices in $\mathbb{R}^n$ with $\mathrm{SL}(n, \mathbb{Z}) \backslash \mathrm{SL}(n, \mathbb{R})$.
- (2) Construction of Siegel domains.
- (3) Mahler's compactness criterion.
- (4) $\mathrm{SL}(n, \mathbb{Z})$ is a lattice!

Ergodicity and mixing properties of flows on homogeneous spaces.

- (1) Review of ergodicity and mixing in measure preserving systems.
- (2) Unitary representations and the Mautner phenomenon
- (3) Howe-Moore Theorem

Divergence properties of unipotent flows.

- (1) The Dani-Kleinbock-Margulis Theorem.
- (2) Existence of minimal subsets for unipotent flows
- (3) Radon measures invariant under unipotent flows

Arithmetic lattices in locally compact groups.

- (1) Groups defined over $\mathbb{Q}$.
- (2) Some cases of Borel Harish-Chandra theorem.
- (3) Borel Density Theorem

The Oppenheim Conjecture.

- (1) Some basics about quadratic forms and statement of the Oppenheim Conjecture
- (2) Translation to dynamics
- (3) Proof of the Oppenheim Conjecture

Action of full diagonal group.

- (1) A conjecture of Cassels-Swinnerton-Dyer
- (2) Translation to dynamics
- (3) Classification of divergent orbits
- (4) Topological entropy and the action of the full diagonal group.

Introduction to Ratner's measure classification of theorem.

- (1) Statement of Ratner's theorems about measure classification, orbit closures, and distribution of individual orbits.
- (2) Proof of some cases of Ratner's measure classification of theorem as time allows.

PREREQUISITES

Prerequisites will be kept to a minimum. We will pursue an aggressively lowbrow approach to algebraic groups, and with respect to ergodic theory will not use much beyond the pointwise ergodic theorem (certainly Ch. 2 of [3] contains more than we will use).

SELECTION OF RELATED BOOKS AND PAPERS

- [1] M. B. Bekka and M. Mayer, *Ergodic theory and topological dynamics of group actions on homogeneous spaces*, London Mathematical Society Lecture Note Series, vol. 269, Cambridge University Press, Cambridge, 2000. MR1781937 (2002c:37002)
- [2] S. G. Dani and G. A. Margulis, *Values of quadratic forms at integral points: an elementary approach*, Enseign. Math. (2) **36** (1990), no. 1-2, 143–174. MR1071418 (91k:11053)
- [3] M. Einsiedler and T. Ward, *Ergodic Theory with a view towards Number Theory*. Springer Graduate Text in Mathematics, to appear, available online for only a bit longer.
- [4] D. W. Morris, *Ratner's theorems on unipotent flows*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 2005. MR2158954 (2006h:37006)