Quantum Spectra and Transport

Michael Aizenman
Resonant Delocalization over Some Infinite and Finite Graphs

Eric Akkermans
Universal Current Fluctuations and Large Deviations in the Symmetric Exclusion Process and Diffusive Systems on an Arbitrary Graph

The purpose of this talk is to show that the statistics of the current of the symmetric simple exclusion process (SSEP) connected to two reservoirs are the same on an arbitrary graph or domain in d dimensions as in the one dimensional case. To prove this result supported by numerical results, we shall use two different approaches. One is based on the macroscopic fluctuation theory of Bertini, De Sole, Gabrielli, Jona-Lasinio and Landim and it uses a direct calculation which bears a close analogy with electrostatics. The second approach makes use of energy forms.

Jean Bellissard
Modeling Liquid Metals and Bulk Metallic Glasses

Properties of bulk metallic glasses and their liquid phase will be reviewed first. Then the cluster model will be presented which describes accurately the short range behavior of these materials. In the third part of this presentation, we will present the mathematical formalism liable to describe these materials at all scale. At last the "anankeon" theory will be presented together with some hint about work in progress.

Michael Berry

Phase-Space Physics: Diabolism, Whorls and Tendrils, Wignereal Disease, Superoscillations...

At the heart of conical refraction – Hamilton's first physical prediction based on phase space – is a conical singularity: a diabolical point. Geometrical objects characterising the classical evolution of families of orbits are phase-space whorls (near stable fixed points) and tendrils (near unstable fixed points). The discordance between chaotic classical evolution and the corresponding quantum evolutions can be understood by the spreading of Wigner functions in phase space: slower as Plancks constant increases. Notwithstanding the uncertainty principle, five equivalent local momenta can be associated with a quantum state, often describing spatial variations much faster than the fourier content of the state might suggest.

Percy Deift

Computing the Eigenvalues of a Random Symmetric Matrix: Universality Results

The speaker will discuss recent numerical results on the computation of the eigenvalues of a random symmetric matrix chosen from various ensembles, using various standard algorithms. A striking feature of these computations is the appearance of universality classes.
This is joint work with Christian Pfrang and Govind Menon.

Jean-Pierre Eckmann

Uniqueness of the Invariant Measure for Networks of Interactions

In the context of non-equilibrium studies of networks of masses connected with springs, there are two issues: Existence and uniqueness of the invariant measure, when the network is driven by by "heat-baths." I will review the question of uniqueness of the invariant measure for such networks under very general conditions on the springs. This is reduced to a question in control theory, and I will explain how one stochastic bath can control several "strands" of masses at once. This is work with Noe Cuneo.

Alexander Elgart

Matrix Valued and Trimmed Anderson Models

Dynamical properties and spectral statistics of the standard one particle Anderson model in the localized regime are well understood by now. However, even slight variations of the model such as addition of the internal degrees of freedom to the particle or trimming of the randomness tend to mud up the entire picture. We will discuss some recent progress concerning such models, established in collaborations with: Abel Klein; Daniel Schmidt; Mira Shamis and Sasha Sodin.

Pavel Exner

The Intriguing δ'

Twenty years ago Yosi brought to my attention interesting features of the one-dimensional singular interaction called, somewhat unfortunately, δ', and their manifestation in the corresponding Wannier-Stark ladder problem. It appeared that there is more to this interaction, and I am going to discuss here some of its other properties. This concerns, in particular, its meaning as it can be understood through appropriate approximations. I will describe also how the δ' manifests itself in coexistence of different spectral types and in spectral properties of singular Schrödinger operators with interactions supported by manifolds of codimension one.

Shmuel Fishman

Anderson Localization for the Nonlinear Schrödinger Equation (NLSE): Results and Puzzles

Martin Fraas

Atomic Clocks: Stationary States and Entropy Production

We describe a new mathematical model of an atomic clock operation and derive various properties of its stationary state. The model consists of classical stochastic equations describing the frequency of the clock coupled (by a measurement and a feedback loop) to a quantum system which describes the atomic reference. Using a Cramer-Rao type inequality we give bounds on the accuracy of the clock in its stationary state and then study the entropy production (of the quantum part due to repeated measurements).

Rupert Frank

Strichartz Inequality for Orthonormal Functions

We prove a Strichartz inequality for a system of orthonormal functions, with an optimal behavior of the constant in the limit of a large number of functions. The estimate generalizes the usual Strichartz inequality, in the same fashion as the Lieb-Thirring inequality generalizes the Sobolev inequality. As an application, we consider the Schr odinger equation in a time-dependent potential and we show the existence of the wave operator in Schatten spaces.
The talk is based on joint work with M. Lewin, E. Lieb and R. Seiringer.

Juerg Froehlich

Information Loss and Entanglement in the Quantum Theory of Measurements

This lecture concerns an application of the theory of open systems to clarifying the foundations of quantum mechanics. I review the quantum theory of experiments and measurements and describe the fundamental roles played by information loss into and entanglement with the environment of a physical system under observation. I discuss an example of repeated non-demolition measurements of a system that result in the emergence of a "fact".

Fritz Gesztesy

Spectral Shift Functions and Some of their Applications in Mathematical Physics

We recall the spectral shift function for (relative) trace class perturbations as developed in detail by M. Krein and some of its applications to Mathematical Physics, including the Witten index (applicable to certain classes of non-Fredholm operators).
We also discuss an extension of the classical spectral shift function to perturbations involving higher trace ideals and derive certain representations for these higher-order spectral shift functions based on modified Fredholm determinants.
This is based on various joint work with A. Carey, Y. Latushkin, K. Makarov, R. Nichols, D. Potapov, F. Sukochev, and Y. Tomilov.

Gian Michele Graf

Bulk-Edge Duality for Topological Insulators

Topological insulators are materials, whose Hamiltonian can not be deformed into that of an ordinary insulator while retaining (fermionic) time-reversal invariance and the insulating property. Indices will be defined for insulators, telling apart the two types. We will do so for two-dimensional insulators: Once based on the electronic bulk states, and once on the edge states. We explain why the indices are equivalent (duality). The result requires only translation invariance of the material in direction of the edge; but in case of full translation invariance the bulk edge can be expressed in the terms of Bloch states. (Joint work with M. Porta)

Italo Guarneri

Inverted Oscillators and Collapse of Wave Packets

A model introduced by Smilansky of a particle and a harmonic oscillator, which have pure point spectra if uncoupled, is known to acquire an absolutely continuous spectrum when they are coupled by a sufficiently strong point interaction. I will expose the dynamical mechanism underlying this spectral phenomenon: the energy of the oscillator exponentially diverges in time, while the spatial probability distribution of the particle collapses into a δ-function at the interaction point. A generalized model with many oscillators coupled to the particle at different points will be argued to provide a formal model for the approximate measurement of position, and collapse of wave packets.

Boris Gutkin

Königsberg Bridges, Periodic Orbits and Ensembles of Truncated Unitary Matrices

Periodic orbits in Hamiltonian systems with chaotic dynamics can be organized into clusters. Orbits from the same cluster traverse through approximately the same points of the phase space and therefore have close actions. Furthermore, the sizes of such clusters can be estimated as a number of Eulerian paths on a graph whose shape depends on the symbolic dynamics of the system. We show that counting of cluster sizes can be turned into a spectral problem for an ensemble of truncated unitary matrices. Based on the conjecture of the universality for the eigenvalues distribution at the spectral edge of these ensembles, we obtain asymptotics of the second moment of cluster distribution. In particular, this result allows us to estimate the average cluster size in a regime of long periodic trajectories.

Ira Herbst

Exponential Decay of Eigenfunctions of Elliptic Partial Differential Operators

For eigenfunctions in the generalized N-body problem of quantum mechanics there is in general a countable closed set of decay rates possible for each eigenvalue. For a fixed N-body Hamiltonian the actual decay rate will vary with the addition of an N-particle interaction potential which decays at infinity. In this joint work with Erik Skibsted, we consider eigenfunctions of higher order self-adjoint partial differential operators on $\mathbb{R}^d$. We look at the simplest case where the only x-dependence is in the zeroth order term which decays at infinity. We determine algebraically the possible decay rates and prove these are the only ones which can occur.

Vojkan Jaksic

Jacobi Matrices and Non-Equilibrium Statistical Mechanics

We consider XY chain with Hamiltonian

$$\frac{1}{2}\sum_{x\in\mathbb{Z}} J_x \left(\sigma_x^{(1)}\sigma_{x+1}^{(1)} + \sigma_x^{(2)}\sigma_{x+1}^{(2)}\right) + \lambda_x\sigma_x^{(3)}$$

where initially the left ($x < 0$)/right ($x > 0$) part of the chain is in thermal equilibrium at inverse temperature β_l/β_r. The temperature differential results in a non-trivial energy/entropy flux across the chain. In this talk I will describe the link between non-equilibrium characteristics of this model and scattering properties of the Jacobi matrix $hu_x = J_x u_{x+1} + \lambda_x u_x + J_{x-1}u_{x-1}$ canonically associated to the XY chain. This talk is based on a joint work with B. Landon and C-A. Pillet.

Svetlana Jitomirskaya

Analytic One-Frequency Cocycles

We show that on a dense open set of analytic quasiperiodic one-frequency matrix cocycles in arbitrary dimension Oseledets filtration is either dominated or trivial. The underlying mechanism is different from that of the Bochi-Viana theorem for continuous cocycles, which links non-domination with discontinuity of the Lyapunov exponent. Indeed, we show that, in our setting, the Lyapunov exponents are jointly continuous in the frequency and theholomorphic matrix valued function of the cocycle, at irrational frequencies. Using approximations by rational frequencies, domination can be characterized by a gap in the Lyapunov spectrum and certain regular behavior of the Lyapunov exponent when extending to complex phases, which in turn always occurs for small nonzero complexifications under the gap condition. It is a report on a joint work with A. Avila and C. Sadel.

Alain Joye

Spectral Transition for Random Quantum Walks on Trees

We consider random quantum walks on a homogeneous tree of degree 3 describing the discrete time evolution of a quantum particle with internal degree of freedom in $\mathbb{C}^3$ hopping on the neighboring sites of the tree in presence of static disorder. The one time step random unitary evolution operator of the particle depends on a unitary matrix $C \in U(3)$ which monitors the strength of the disorder. We show the existence of open sets of matrices in $U(3)$ for which the random evolution has either pure point spectrum almost surely or purely absolutely continuous spectrum. We also establish properties of the spectral diagram which provide a description of the spectral transition driven by $C \in U(3)$.

Abel Klein

Unique Continuation Principle for Spectral Projections of Schrödinger Operators and Optimal Wegner Estimates for Non-Ergodic Random Schrödinger Operators

We prove a unique continuation principle for spectral projections of Schrödinger operators. We consider a Schrödinger operator $H = -\Delta + V$ on $\mathrm{L}^2(\mathbb{R}^d)$, and let H_Λ denote its restriction to a finite box Λ with either Dirichlet or periodic boundary condition. We prove unique continuation estimates of the type $\chi_I(H_\Lambda) W \chi_I(H_\Lambda) \geq \kappa\, \chi_I(H_\Lambda)$ with $\kappa > 0$ for appropriate potentials $W \geq 0$ and intervals I. As an application, we obtain optimal Wegner estimates at all energies for a class of non-ergodic random Schrödinger operators with alloy-type random potentials ('crooked' Anderson Hamiltonians). We also prove optimal Wegner estimates at the bottom of the spectrum with the expected dependence on the disorder (the Wegner estimate improves as the disorder increases), a new result even for the usual (ergodic) Anderson Hamiltonian. These estimates are applied to prove localization at high disorder for Anderson Hamiltonians in a fixed interval at the bottom of the spectrum.

Israel Klich

From Frustration to Quantum Glassiness via Exotic Entropy Scaling

When spins are regularly arranged in a triangular fashion, the spins may not satisfy simultaneously their antiferromagnetic interactions with their neighbors. This phenomenon, called frustration, usually leads to a large set of ground states and to exotic states such as spin ice and spin liquid. Here we report a novel situation: a system that freezes into a glass like state induced by quantum fluctuations, in contrast to the known mechanisms for classical spin-glass which rely on the presence of disorder. At the heart of the effect is an unusual scaling of the number of local minima, with a scaling extensive in the boundary length rather than the volume, which we describe in terms of a new tiling problem. This fascinating system gives rise to many mathematical and physical questions, and possibly provides a new paradigm for glassiness through frustration without disorder.

Netanel Lindner

New Platforms for Topological Quantum Computing

Motivated by recent proposals for creating Majorana Fermion zero modes by appropriately coupling readily available materials, we consider the possibility of engineering new non-Abelian systems from Abelian components. In particular, we study the non-Abelian statistics characterizing edges of Abelian fractional quantum Hall states, when proximity coupled to superconductors and ferromagnets. We show that as more superconductor-ferromagnet interfaces are introduced, the ground state degeneracy grows with a quantum dimension of a square root of an even integer, corresponding to a new type of non-Abelian statistics. Topologically protected braiding of two interfaces can be achieved by a sequence of adiabatic manipulations of the system. We discuss the general framework describing the non-Abelian statistics realized in such systems, and explain how it differs from the conventional theory of anyons. We show that non-Abelian representations of the braid group realized in these systems are richer then those realized by non-Abelian anyons based on Majorana fermions. We discuss implications to topological quantum computation, and consider possible realizations of these ideas in experimentally accessible solid state systems.

Claude-Alain Pillet

Entropic Fluctuations of Quantum Dynamical Semigroups

We study finite dimensional quantum dynamical semigroups whose generator L is a sum of Lindbladians satisfying the detailed balance condition. Such semigroups arise in the weak coupling (Van Hove) limit of Hamiltonian dynamical systems describing open quantum systems out of equilibrium. We prove a general entropic fluctuation theorem for this class of semigroups by relating the cumulant generating function of entropy transport to the spectrum of a family of deformations of the generator L. We show that, besides the celebrated Evans-Searles symmetry, this cumulant generating function also satisfies the translation symmetry recently discovered by Andrieux et al., and that in the linear regime near equilibrium these two symmetries yield Kubos and Onsagers linear response relations. This is joint work with V. Jaksic and M. Westrich.

Jonathan Robbins

Quantum Statistics on Graphs

We consider quantum statistics for indistinguishable particles without spin on a one-dimensional network, or graph. In spite of the fact that graphs are locally one-dimensional, anyon statistics can emerge in a generalized form. The quantum statistics depends on where and how the particles are exchanged, and discrete sign-valued phases appear alongside continuous anyon and Aharonov-Bohm phases.
It turns out that there is a complete characterisation of abelian statistics in terms of the topology of the graph, in which connectedness plays a key role. For three-connected graphs, the possible statistics are restricted to anyon or Bose/Fermi, according to whether the graph is planar or not. New possibilities appear for two-connected graphs, although the statistics remains independent of the number of particles (so an afbau principle applies). The greatest complexity appears for one-connected graphs, for which the statistics can depend on the number of particles as well. The results are illustrated with examples. The analysis is simplified by considering discrete rather than metric graphs.
This is joint work with Jon Harrison, Jon Keating and Adam Sawicki.

Lorenzo Sadun

Statistical Mechanics of Large Random Graphs

Consider a large graph with n vertices where the number of actual edges is a fraction e of the $\binom{n}{2}$ possible edges, and the number of triangles is a fraction t of the $\binom{n}{3}$ possible triangles. If we fix the values of e and t, what are the likeliest structures for the graph? In the $n \to \infty$ limit this becomes a problem in statistical mechanics, akin to the microcanonical ensemble. The likeliest structures depend on e and t, and there are a number of interesting phase transitions in the (e, t) plane.

Hermann Schulz-Baldes

Invariants for J-Unitaries on Real Krein Spaces and the Classification of Transfer Operators

Essentially S^1-gapped J-unitaries on a Krein space (K, J) conserving, moreover, a Real structure, are shown to possess Z or Z_2-valued homotopy invariants which can be expressed in terms of Krein inertia of the eigenvalues on the unit circle and are rooted in a detailed analysis of their collisions. When applied to the transfer operators associated with periodic two-dimensional tight-binding Hamiltonians, these new invariants determine the existence and nature of the surface modes (Majorana or conventional fermions, chirality) and allow to distinguish different phases of various topological insulators.

Ruedi Seiler

Universally Typical Sets for Ergodic Sources of Multidimensional Data

We lift important results of the theory of samples of discrete ergodic information sources to the multidimensional setting. We use the technique of packings and coverings with multidimensional windows in entropy estimation and universal lossless compression. In particular, we construct sequences of multidimensional array sets which, in the limit, build the generated samples of any ergodic source of entropy rate below an h0 with probability 1 and whose cardinality grows at most at exponential rate h0. Thereby we extrapolate mathematical framework relevant for universal source coding of multidimensionally correlated data.

Barry Simon

Yosi and Me

I'll review some of the joint work of Yosi Avron and B. Simon

Uzy Smilansky

Tournaments and their Spectra

A round robin tournament is a competition between N players, each plays once against all the rest and the outcome of a game is definite, (no draw). The outcome can be summarized in an NxN matrix with the (i,j) entry being 1 if player i wins and 0 otherwise. One can also consider these matrices as directed graph. The spectra of tournament matrices show interesting features, and their statistics can be studies when one studies the entire ensemble of NxN tournaments endowed with uniform probability. No wonder - Random Matrix theory reproduces the observed statistical features, which can be addressed by using a trace formula connecting the spectra to the counting statistics of cycles on the directed graph.

Stefan Teufel

Adiabatic Theorems for Particles Coupled to Massless Fields

I review some older and some new results on adiabatic theorems for particles coupled to massless fields. They all relate in some way or other to the adiabatic theorem without gap condition of Avron and Elgart. As my main example I will discuss spontaneous emission of photons during molecular reactions.