תרגיל מס'1 בחדו"א 2 למהנדסים.

<u>הנושא: האינטגרל הלא מסויים.</u>

פתרונות ותשובות.

א)

$$\int \frac{2x+3}{x^2+x-2}dx = \int \frac{2x+1}{x^2+x-2}dx + \int \frac{2}{x^2+x-2}dx = \left| t = x^2+x-2 \;\; dt = (2x+1)dx \right| =$$

$$= \int \frac{dt}{t} + + \int \frac{2dx}{\left(x+\frac{1}{2}\right)^2 + \frac{9}{4}} = \ln\left|x^2+x-2\right| + \frac{2}{3}\arctan\left(\frac{2}{3}\cdot\left(x+\frac{1}{2}\right)\right) + C$$

ב)

$$\int \frac{1}{3x^2-2x-5}dx = \frac{1}{3}\int \frac{1}{x^2-\frac{2}{3}x-\frac{5}{3}}dx + \int \frac{1}{\left(x-\frac{1}{3}\right)^2-\frac{16}{9}}dx = -\frac{3}{8}\ln\left|\frac{\frac{4}{3}+\left(x-\frac{1}{3}\right)}{\frac{4}{3}-\left(x-\frac{1}{3}\right)}\right| =$$

$$= -\frac{3}{8}\ln\left|\frac{1+x}{\frac{5}{3}+x}\right| + C$$

ג) $$\int \frac{\sqrt{\ln x}}{x}dx = \left| t = \ln x \, dt = \frac{dx}{x} \right| = \int \sqrt{t}dt = \frac{2}{3}(\ln x)^{\frac{3}{2}} + C$$

ד)

$$\int \cos^3 x dx = \int \left(1-\sin^2 x\right)\cos x dx = \left| t = \sin x \;\; dt = \cos x dx \right| = \int \left(1-t^2\right)dt = t - \frac{t^3}{3} =$$

$$= \sin x - \frac{\sin^3 x}{3} + C$$

ה) $$\int \frac{e^x}{e^x+3}dx = \left| t = e^x+3 \;\;\; dt = e^x dx \right| = \int \frac{1}{t}dt = \ln\left(e^x+3\right) + C$$

ו) $$\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}dx = \left| t = \arcsin x \;\; dt = \frac{1}{\sqrt{1-x^2}}dx \right| = \int e^t dt = e^{\arcsin x} + C$$

ז) $$\int x^3 \ln x dx = \left| \begin{matrix} u = \ln x & v' = x^3 \\ u' = \frac{1}{x} & v = \frac{x^4}{4} \end{matrix} \right| = \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

ח) $$\int \frac{x}{\sqrt{x^2+5}}dx = \left| t = x^2+5 \;\; dt = 2xdx \right| = \int \frac{dt}{2\sqrt{t}} = \sqrt{t} = \sqrt{x^2+5} + C$$

ט) $$\int \frac{\cos x}{1+\sin^2 x}dx = \left| t = \sin x \;\; dt = \cos x dx \right| = \int \frac{dt}{1+t^2} = \arctan\left(\sin^2 x\right) + C$$

ח*)

$$\int x^2\ln\frac{1-x}{1+x}dx=\int x^2\ln(1-x)dx-\int x^2\ln(1+x)dx=\begin{vmatrix} u=\ln(1-x)\ v'=x^2 & u=\ln(1+x)\ v'=x^2 \\ u'=\frac{-1}{(1-x)}\quad v=\frac{x^3}{3} & u'=\frac{1}{1+x}\quad v=\frac{x^3}{3}\end{vmatrix}=$$

$$=\frac{x^3}{3}\ln(1-x)+\frac{1}{3}\int\frac{x^3}{1-x}dx+\frac{x^3}{3}\ln(1+x)-\frac{1}{3}\int\frac{x^3}{1+x}dx=$$

$$=\frac{x^3}{3}\ln((1-x)(1+x))++\frac{1}{3}\left(\int\left(-x^2-x-1+\frac{1}{1-x}\right)dx-\int\left(x^2-x+1-\frac{1}{1+x}\right)dx\right)=$$

$$=\frac{1}{3}\left(2\ln((1-x)(1+x))-\frac{2}{3}x^3-2x\right)+C$$

(י)

$$\int\frac{\ln(\sin x)}{\sin^2 x}dx=\begin{vmatrix} u=\ln(\sin x)\ \ v'=\frac{1}{\sin^2 x} \\ u'=\frac{\cos x}{\sin x}\quad v=-\cot x\end{vmatrix}=$$

$$=-\cot x\ln(\sin x)+\int\frac{\cos^2 x}{\sin^2 x}dx=-\cot x\ln(\sin x)+\int\frac{1}{\sin^2 x}dx-\int dx=$$

$$=-\cot x\ln(\sin x)-\cot x-x+C$$

(יב)

$$\int\cos^6 xdx=\int(\cos^2 x)^2\cos^2 xdx=\frac{1}{8}\int(1+\cos 2x)^2(1+\cos 2x)dx=\frac{1}{8}$$

$$\int(1+2\cos 2x+\cos^2 2x)(1+\cos 2x)dx=\int\frac{1}{8}\int(1+3\cos 2x+3\cos^2 2x+\cos^3 2x)=$$

$$\int\frac{1}{8}\left(1+3\cos 2x+\frac{3}{2}+\frac{3}{2}\cos 4x+(1-\sin^2 2x)\cos 2x\right)dx=$$

$$=\frac{1}{8}\left(x+\frac{3}{2}\sin 2x+\frac{3}{8}\sin 4x+\int(1-\sin^2 2x)\cos 2xdx\right)=$$

$$=\left|t=\sin 2x\ \ dt=\frac{1}{2}\cos 2x\right|=\frac{1}{8}\left(x+\frac{3}{2}\sin 2x+\frac{3}{8}\sin 4x+2\left(\sin 2x-\frac{\sin^3 x}{3}\right)\right)+C$$

(יג)

$$\int\tan^5 xdx=\int\frac{\sin^5 x}{\cos^5 x}dx=\int\frac{(1-\cos^2 x)^2\sin x}{\cos^5 x}dx=|t=\cos x\ \ dt=-\sin xdx|=-\int\frac{(1-t^2)^2dt}{t^5}=$$

$$=-\left(\int\frac{1}{t^5}dt-2\int\frac{dt}{t^3}+\int\frac{dt}{t}\right)=\frac{1}{4t^4}-\frac{1}{t^2}-\ln|t|=\frac{1}{4\cos^4 x}-\frac{1}{\cos^2 x}-\ln|\cos x|+C$$

(טו)

$$\int\sin^2 x\cos^3 xdx=\int\sin^2 x(1-\sin^2 x)\cos xdx=|t=\sin x\ \ dt=\cos xdx|=\int t^2(1-t^2)dt=\frac{t^3}{3}-\frac{t^5}{5}=$$

$$=\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$$

(טז)

$$\int \frac{dx}{\cos x} = \left| \begin{array}{l} t = \tan\frac{x}{2} \Rightarrow x = ac\tan 2t \Rightarrow dx = \frac{2}{1+4t^2}dt \\ \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1-t^2}{1+t^2} \end{array} \right| = 2\int \frac{1+t1^2}{\left(1-t^2\right)\left(1+4t^2\right)}dt =$$

$$.= 2\left(\int \frac{1}{5}\frac{1}{1-t}dt + \frac{1}{5}\int \frac{1}{1+t}dt + \frac{3}{5}\int \frac{1}{1+4t^2}dt\right) = 2\left(-\frac{1}{5}\ln|1-t| + \frac{1}{5}\ln|1+t| + \frac{1}{8}\arctan\frac{t}{2}\right) =$$

$$= 2\left(\frac{1}{5}\ln\left|\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right| + \frac{1}{8}\arctan\left(\frac{1}{2}\tan\frac{x}{2}\right)\right) + C$$

בהצלחה!