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The Forgetful Passenger*

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When there is absent-mindedness, probabilities may change even when no new information becomes available. A similar phenomenon occurs in general imperfect recall situations. *Journal of Economic Literature* Classification Numbers: D81, C72.

An automobile with a passenger starts at START in Fig. 1. At X it exits with probability $\frac{1}{2}$ and continues with probability $\frac{1}{2}$. At Y, again, it exits with probability $\frac{1}{2}$ and continues with probability $\frac{1}{2}$. The passenger has no control over the automobile. Moreover, he cannot distinguish between intersections X and Y and cannot remember whether he has already gone through one of them.

At START, the passenger's probability for arriving at C is $\frac{1}{4}$. At X, it is[1] $\frac{1}{3}$. But in moving from START to X, the passenger has received no signal, no new information. At START, he knew with probability 1—indeed, with absolute certainty—that he will arrive at X. How, then, can his probability change? But it does!

The apparent paradox can be cast in decision-theoretic terms as follows: At each node (START, X, and Y), the passenger is offered, for \$30, a lottery ticket that yields \$100 if C is reached, and \$0 otherwise. The ticket yields an expected loss of \$5 at START and an expected gain of $\$3\frac{1}{3}$ at X. Assuming linear utilities, therefore, he would refuse at START and accept at X; and this even though he gets from START to X with *certainty*.

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[1] At X, he knows he is at X or Y, and his probability p_X for being at X is twice his probability p_Y for being at Y (since he goes through X with probability 1, and through Y with probability $\frac{1}{2}$). Therefore, $p_X = \frac{2}{3}$ and $p_Y = \frac{1}{3}$ (see also Footnote 3). So at X (and at Y), his probability for C is $(\frac{2}{3}) \cdot (\frac{1}{2})^2 + (\frac{1}{3}) \cdot (\frac{1}{2}) = \frac{1}{3}$.

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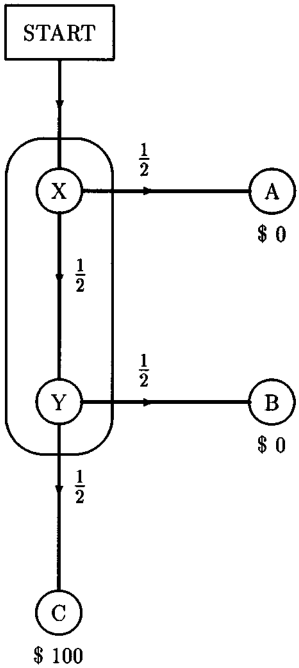

FIG. 1. The absent-minded passenger.

How can this be? In particular, since he *knows* at START that he will get to X, and that he will buy there, why shouldn't he buy at START?

The reader may want to stop here and try to figure it out.

* * * * *

The answer is simple. The two intersections, X and Y, are indistinguishable. When the passenger buys a lottery ticket at X, he does not know that he is at X. As far as he knows, he may be at Y—in which case the lottery is favorable. In contrast, "being at START" is like "being at X for sure"—and the lottery is unfavorable there.

To clarify this further, consider the overall payoff expected from the decision "at an intersection, buy a ticket." (Recall that the two intersections are indistinguishable, and thus a ticket will be bought at each intersection through which the car goes.) With probability $\frac{1}{2}$, the passenger will end up at A, with one ticket and no prize; with probability $\frac{1}{4}$, he will end up at B, with two tickets and no prize; with probability $\frac{1}{4}$, he will end up at C, with two tickets and two prizes of \$100 each. The expected value is therefore $(\frac{1}{2}) \cdot (-\$30) + (\frac{1}{4}) \cdot (-\$60) + (\frac{1}{4}) \cdot (\$200 - \$60) = \$5$—which

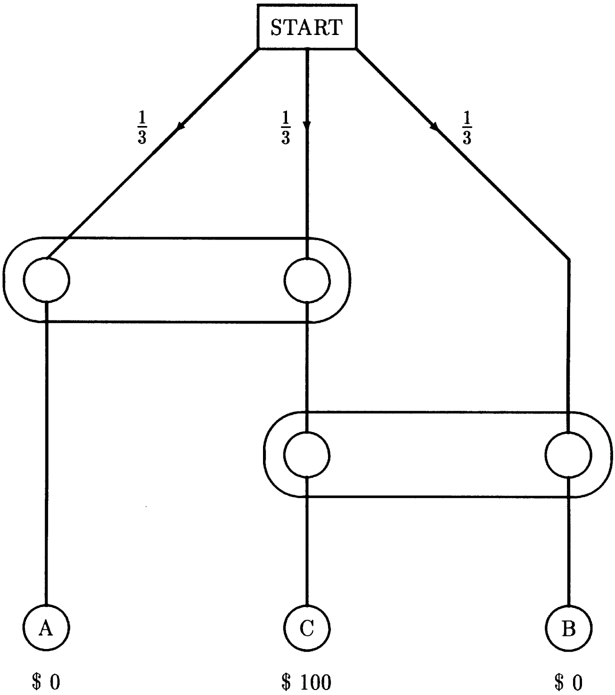

FIG. 2. The forgetful passenger.

is positive! What happens is that at Y, the lottery ticket is "good"; at X—which is the same as at START—it is "bad." In balance, it turns out to be good.[2,3]

Note that this change in behavior has nothing to do with the "absent-minded driver" (Piccione and Rubinstein, 1997). The issue there is to compare what one *plans* at START (to do at the intersections) with what one *does* at the intersections. As pointed out in Aumann, Hart, and Perry (1997), the decision that is optimal there at the planning stage is also optimal at the action stage. This is a general phenomenon. In particular, it holds for the passenger in the current example: the planning optimal decision is to buy a lottery ticket at each intersection, and this is also the action optimal decision.[4]

[2] Thus one should *not* buy tickets at START, since one wants to have as few tickets as possible at X, and as many as possible at Y.

[3] A computation like the one above shows that the price of a fair ticket is $\$33\frac{1}{3}$. When evaluated at an intersection, the ticket is still fair; this implies that the probability that the intersection is X (Y) must indeed be $\frac{2}{3}$ ($\frac{1}{3}$, respectively).

[4] Actually, buying a lottery ticket at the current intersection is a strictly dominant decision, regardless of what is done at the other intersection.

But the issue in this note is entirely different. It is to compare what one *does* (not plans!) at START with what one does at the intersections. As we have seen, these *are* different.

Figure 2 exhibits a similar example that does not involve ‘‘absent-mindedness,’’[5] but only ‘‘forgetfulness’’ (imperfect recall). At START, the passenger's probability for arriving at C is $\frac{1}{3}$, and subsequently it is *certain* to become $\frac{1}{2}$. In terms of lottery tickets, a $100 prize at C is thus worth 33\frac{1}{3}$ at START and $50 at each one of the two information sets—which will surely be reached.

To summarize: Absent-mindedness and imperfect recall, while interesting, entail no time inconsistency or paradox. Aumann, Hart, and Perry (1997) shows that one should do what one planned to do. Here we show that there *is* a difference between what one should do at START and at an intersection.

REFERENCES

Aumann, R. J., Hart, S., and Perry, M. (1997). ‘‘The Absent-Minded Driver,’’ *Games and Econ. Behav.* **20**, 102–116.

Piccione, M., and Rubinstein, A. (1997). ‘‘On the Interpretation of Decision Problems with Imperfect Recall,’’ *Games and Econ. Behav.* **20**, 3–24.

[5] where a play intersects an information set more than once (Piccione and Rubinstein, 1997, end of Section 3).