Correlated Equilibria: Rationality and Dynamics

Sergiu Hart

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AUMANN 80
CORRELATED EQUILIBRIA: RATIONALITY AND DYNAMICS

Sergiu Hart

Center for the Study of Rationality
Dept of Mathematics    Dept of Economics
The Hebrew University of Jerusalem

hart@huji.ac.il
http://www.ma.huji.ac.il/hart
Correlated Equilibrium

Correlated equilibrium is a generalization of Nash equilibrium in game theory, in which players have the opportunity to coordinate their strategies through a trusted third party. This coordination can lead to outcomes that are more efficient than those achievable through Nash equilibrium alone.

Aumann, JME 1974
Correlated Equilibrium

CORRELATED EQUILIBRIUM:

Nash equilibrium when players receive payoff-irrelevant information before playing the game

Aumann, JME 1974
A Correlated Equilibrium is a Nash equilibrium when players receive payoff-irrelevant signals before playing the game.
A **Correlated Equilibrium** is a Nash equilibrium when players receive payoff-irrelevant signals before playing the game

- **Examples:**
A Correlated Equilibrium is a Nash equilibrium when players receive payoff-irrelevant signals before playing the game.

- **Examples:**
  - Independent signals
A **Correlated Equilibrium** is a Nash equilibrium when players receive payoff-irrelevant signals before playing the game.

**Examples:**
- Independent signals $\iff$ Nash equilibrium
A Correlated Equilibrium is a Nash equilibrium when players receive payoff-irrelevant signals before playing the game.

Examples:

- Independent signals $\iff$ Nash equilibrium
- Public signals (“sunspots”)

Correlated Equilibrium

SERGIU HART © 2010 – p. 4
A **Correlated Equilibrium** is a Nash equilibrium when players receive payoff-irrelevant signals before playing the game

- **Examples:**
  - Independent signals $\iff$ Nash equilibrium
  - Public signals ("sunspots") $\iff$ convex combinations of Nash equilibria
A Correlated Equilibrium is a Nash equilibrium when players receive payoff-irrelevant signals before playing the game.

**Examples:**
- Independent signals ⇔ Nash equilibrium
- Public signals (“sunspots”) ⇔ convex combinations of Nash equilibria
- Butterflies play the Chicken Game (“Speckled Wood” *Pararge aegeria*)
## "Chicken" game

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<thead>
<tr>
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"Chicken" game

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A Nash equilibrium
Correlated Equilibria

"Chicken" game

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another Nash equilibrium
## Correlated Equilibria

### "Chicken" game

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A (publicly) correlated equilibrium
"Chicken" game

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<td>LEAVE</td>
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<td>1/3</td>
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<tr>
<td>STAY</td>
<td>1/3</td>
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Another correlated equilibrium

- After signal L play LEAVE
- After signal S play STAY
A Correlated Equilibrium is a Nash equilibrium when the players receive payoff-irrelevant signals before playing the game (Aumann 1974)

Examples:
- Independent signals $\iff$ Nash equilibrium
- Public signals ("sunspots") $\iff$ convex combinations of Nash equilibria
- Butterflies play the Chicken Game ("Speckled Wood" Pararge aegeria)
A Correlated Equilibrium is a Nash equilibrium when the players receive payoff-irrelevant signals before playing the game (Aumann 1974)

- **Examples:**
  - Independent signals $\iff$ Nash equilibrium
  - Public signals (“sunspots”) $\iff$ convex combinations of Nash equilibria
  - Butterflies play the Chicken Game (“Speckled Wood” *Pararge aegeria*)
  - Boston Celtics (NBA)
Correlated Equilibrium

Signals (public, correlated) are **unavoidable**
Correlated Equilibrium

- Signals (public, correlated) are **unavoidable**
- **Common Knowledge** of **Rationality** $\iff$ **Correlated Equilibrium** (Aumann 1987)
Correlated Equilibrium

- Signals (public, correlated) are **unavoidable**

- **Common Knowledge of Rationality** ⇔ **Correlated Equilibrium** (Aumann 1987)

A joint distribution \( z \) is a **correlated equilibrium**

\[
\sum_{s^{-i}} u(j, s^{-i}) z(j, s^{-i}) \geq \sum_{s^{-i}} u(k, s^{-i}) z(j, s^{-i})
\]

for all \( i \in N \) and all \( j, k \in S^i \)
FACT
FACT

There are no general, natural dynamics leading to Nash equilibrium
FACT

There are no general, natural dynamics leading to Nash equilibrium

"general"
FACT

There are no general, natural dynamics leading to Nash equilibrium

"general": in all games
FACT

There are no general, natural dynamics leading to Nash equilibrium

"general" : in all games rather than: in specific classes of games
FACT

There are no general, natural dynamics leading to Nash equilibrium

"general": in all games rather than: in specific classes of games:
- two-person zero-sum games
- two-person potential games
- supermodular games
- . . .
FACT

There are no general, natural dynamics leading to Nash equilibrium
FACT

There are no general, natural dynamics leading to Nash equilibrium

"leading to Nash equilibrium"
FACT

There are no general, natural dynamics leading to Nash equilibrium

"leading to Nash equilibrium": at a Nash equilibrium (or close to it) from some time on
FACT

There are no general, natural dynamics leading to Nash equilibrium
FACT

There are no general, natural dynamics leading to Nash equilibrium

"natural"
Dynamics

FACT

There are no general, natural dynamics leading to Nash equilibrium

"natural":
FACT

There are no general, *natural* dynamics leading to Nash equilibrium

- "natural":
  - adaptive (reacting, improving, ...)
FACT

There are no general, natural dynamics leading to Nash equilibrium

"natural":
- adaptive (reacting, improving, ...)
- simple and efficient
There are no general, \textit{natural} dynamics leading to Nash equilibrium

"\textit{natural}":
- adaptive (reacting, improving, ...)
- simple and efficient:
  - computation (performed at each step)
Dynamics

**FACT**

*There are no general, natural dynamics leading to Nash equilibrium*

"natural":

- **adaptive** (reacting, improving, ...)
- **simple and efficient**:
  - **computation** (performed at each step)
  - **time** (how long to reach equilibrium)
FACT

There are no general, natural dynamics leading to Nash equilibrium

"natural":

- adaptive (reacting, improving, ...)
- simple and efficient:
  - computation (performed at each step)
  - time (how long to reach equilibrium)
  - information (of each player)
FACT

There are no general, natural dynamics leading to Nash equilibrium

"natural":

- adaptive (reacting, improving, ...)
- simple and efficient:
  - computation (performed at each step)
  - time (how long to reach equilibrium)
  - information (of each player)

bounded rationality
Dynamics that are NOT "natural":
Dynamics that are **NOT** "natural":

- **exhaustive search**
  (deterministic or stochastic)
Exhaustive Search
Exhaustive Search

\[ E = m \alpha^2 \]
Exhaustive Search
Exhaustive Search
Exhaustive Search

\[ E = mc^2 \]

\[ E = m \cdot b^2 \]
Exhaustive Search

\[ E = m \alpha^2 \]
\[ E = m \beta^2 \]
\[ E = mc^2 \]
Exhaustive Search

\[ E = mc^2 \]

\[ E = ma^2 \]

\[ E = mb^2 \]
Exhaustive Search
Dynamics

Dynamics that are **NOT** "natural":

- **exhaustive search**
  (deterministic or stochastic)
Dynamics that are NOT "natural":

- exhaustive search (deterministic or stochastic)
- using a mediator
Dynamics

Dynamics that are **NOT** "natural":

- **exhaustive search**
  (deterministic or stochastic)
- using a **mediator**
- **broadcasting** the private information
  and then performing **joint** computation
Dynamics

Dynamics that are NOT "natural":

- **exhaustive search**
  (deterministic or stochastic)

- using a **mediator**

- **broadcasting** the private information
  and then performing **joint** computation

- **fully rational learning**
  (prior beliefs on the strategies of the opponents, Bayesian updating, optimization)
FACT

There are no general, natural dynamics leading to Nash equilibrium

"natural":

- adaptive

simple and efficient:
- computation (performed at each step)
- time (how long to reach equilibrium)
- information (of each player)
Dynamics

FACT

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FACT

There are no general, natural dynamics leading to Nash equilibrium

"natural":

- adaptive
- simple and efficient:
  - computation (performed at each step)
  - time (how long to reach equilibrium)
  - information (of each player)
Each player knows *only* his own payoff (utility) function.
Each player knows *only* his own payoff (utility) function

*(does not know the payoff functions of the other players)*
Each player knows \textit{only} his own payoff (utility) function

\textit{(does not know the payoff functions of the other players)}

\textit{Hart and Mas-Colell, AER 2003}
UNCOUPLED DYNAMICS:

Each player knows *only* his own payoff (utility) function

(does *not* know the payoff functions of the other players)

(privacy-preserving, decentralized, distributed ...)

Hart and Mas-Colell, AER 2003
$N$-person game in strategic (normal) form:

Players

\[ i = 1, 2, ..., N \]
$N$-person game in strategic (normal) form:

- **Players**

  \[ i = 1, 2, \ldots, N \]

- **For each player $i$: Actions**

  \[ a^i \text{ in } A^i \]
$N$-person game in strategic (normal) form:

- **Players**
  
  \[ i = 1, 2, \ldots, N \]

- For each player $i$: **Actions**
  
  \[ a^i \text{ in } A^i \]

- For each player $i$: **Payoffs (utilities)**
  
  \[ u^i(a) \equiv u^i(a^1, a^2, \ldots, a^N) \]
Time

\[ t = 1, 2, \ldots \]
Dynamics

- **Time**

\[ t = 1, 2, \ldots \]

- At period \( t \) each player \( i \) chooses an action \( a_t^i \) in \( A^i \)
Dynamics

- Time
  
  \[ t = 1, 2, \ldots \]

- At period \( t \) each player \( i \) chooses an action
  
  \( a_t^i \text{ in } A^i \)

  according to a probability distribution

  \( \sigma_t^i \text{ in } \Delta(A^i) \)
Dynamics

Fix the set of players $1, 2, \ldots, N$ and their action spaces $A^1, A^2, \ldots, A^N$
Fix the set of players 1, 2, ..., \( N \) and their action spaces \( A^1, A^2, ..., A^N \)

A general dynamic:
Dynamics

Fix the set of players $1, 2, \ldots, N$ and their action spaces $A^1, A^2, \ldots, A^N$

- A general dynamic:

\[ \sigma^i_t \equiv \sigma^i_t ( \text{HISTORY} ; \text{GAME} ) \]
Fix the set of players 1, 2, ..., \( N \) and their action spaces \( A^1, A^2, ..., A^N \)

A general dynamic:

\[
\sigma_t^i \equiv \sigma_t^i ( \text{HISTORY} ; \text{GAME} )
\]

\[
\equiv \sigma_t^i ( \text{HISTORY} ; u^1, ..., u^i, ..., u^N )
\]
Uncoupled Dynamics

Fix the set of players $1, 2, \ldots, N$ and their action spaces $A^1, A^2, \ldots, A^N$

A general dynamic:

$$\sigma^i_t \equiv \sigma^i_t (\text{HISTORY} ; \text{GAME})$$

$$\equiv \sigma^i_t (\text{HISTORY} ; u^1, \ldots, u^i, \ldots, u^N)$$

An **UNCOUPLLED** dynamic:
Uncoupled Dynamics

Fix the set of players $1, 2, \ldots, N$ and their action spaces $A^1, A^2, \ldots, A^N$

- A general dynamic:

$$\sigma^i_t \equiv \sigma^i_t (\text{HISTORY} ; \text{GAME})$$

$$\equiv \sigma^i_t (\text{HISTORY} ; u^1, \ldots, u^i, \ldots, u^N)$$

- An UNCOUPLED dynamic:

$$\sigma^i_t \equiv \sigma^i_t (\text{HISTORY} ; u^i)$$
How long to equilibrium?
HOW LONG TO EQUILIBRIUM?

Estimate the number of time periods it takes until a Nash equilibrium is reached.
Natural Dynamics: Time

**HOW LONG TO EQUILIBRIUM?**

Estimate the number of time periods it takes until a Nash equilibrium is reached

How?
HOW LONG TO EQUILIBRIUM?

Estimate the number of time periods it takes until a Nash equilibrium is reached

- How?
- An uncoupled dynamic
  \[ \approx \]
  A distributed computational procedure
HOW LONG TO EQUILIBRIUM?

Estimate the number of time periods it takes until a Nash equilibrium is reached

How?

- An uncoupled dynamic
  \[\approx\]
  A distributed computational procedure

\[\Rightarrow\] COMMUNICATION COMPLEXITY
An uncoupled dynamic leading to Nash equilibria is **TIME-EFFICIENT** if
How Long to Equilibrium

An uncoupled dynamic leading to Nash equilibria is **TIME-EFFICIENT** if the **TIME IT TAKES**
An uncoupled dynamic leading to Nash equilibria is **TIME-EFFICIENT** if the **TIME IT TAKES** is **POLYNOMIAL** in the number of players.
An uncoupled dynamic leading to Nash equilibria is **TIME-EFFICIENT** if the **TIME IT TAKES** is **POLYNOMIAL** in the number of players (rather than: *exponential*)
An uncoupled dynamic leading to Nash equilibria is **TIME-EFFICIENT** if the **TIME IT TAKES** is **POLYNOMIAL** in the number of players (rather than: **exponential**)

**Theorem.** *There are NO TIME-EFFICIENT uncoupled dynamics that reach a pure Nash equilibrium in all games where such equilibria exist.*
An uncoupled dynamic leading to Nash equilibria is **TIME-EFFICIENT** if the **TIME IT TAKES** is **POLYNOMIAL** in the number of players (rather than: exponential)

**Theorem.** *There are NO TIME-EFFICIENT uncoupled dynamics that reach a pure Nash equilibrium in all games where such equilibria exist.*

---

Hart and Mansour, GEB 2010
An uncoupled dynamic leading to Nash equilibria is **TIME-EFFICIENT** if the **TIME IT TAKES** is **POLYNOMIAL** in the number of players (rather than: **exponential**)

**Theorem.** There are **NO TIME-EFFICIENT** uncoupled dynamics that reach a pure Nash equilibrium in all games where such equilibria exist.

In fact: **exponential**, like exhaustive search

---

Hart and Mansour, GEB 2010
FACT

There are NO general, natural dynamics leading to Nash equilibrium
FACT
There are NO general, natural dynamics leading to Nash equilibrium

RESULT
There CANNOT BE general, natural dynamics leading to Nash equilibrium
RESULT

There CANNOT BE general, natural dynamics leading to Nash equilibrium
RESULT

There CANNOT BE general, natural dynamics leading to Nash equilibrium

Perhaps we are asking too much?
RESULT

There CANNOT BE general, natural dynamics leading to Nash equilibrium

Perhaps we are asking too much?

For instance, the size of the data (the payoff functions) is exponential rather than polynomial in the number of players.
RESULT

THERE EXIST *general, natural dynamics* leading to *CORRELATED EQUILIBRIA*
RESULT

THERE EXIST *general, natural dynamics*

*leading to* **CORRELATED EQUILIBRIA**

- **Regret Matching**

---

*Hart and Mas-Colell, Ec’ca 2000*
RESULT

THERE EXIST general, natural dynamics leading to CORRELATED EQUILIBRIA

- Regret Matching
- General regret-based dynamics

Hart and Mas-Colell, Ec’ca 2000, JET 2001
"REGRET": the increase in past payoff, if any, if a different action would have been used.
Regret Matching

- "REGRET": the increase in past payoff, if any, if a different action would have been used

- "MATCHING": switching to a different action with a probability that is proportional to the regret for that action
THERE EXIST *general, natural dynamics* leading to **CORRELATED EQUILIBRIA**
Dynamics & Correlated Equilibria

THERE EXIST **general**, **natural dynamics** leading to **CORRELATED EQUILIBRIA**

- "**general**": in all games
THERE EXIST general, natural dynamics leading to CORRELATED EQUILIBRIA

- "general": in all games
- "natural":

THERE EXIST *general, natural* dynamics leading to CORRELATED EQUILIBRIA

- *"general"*: in all games
- *"natural"*:
  - **adaptive** (also: close to "behavioral")
THERE EXIST *general, natural dynamics* leading to *CORRELATED EQUILIBRIA*

- *"general":* in all games
- *"natural":*
  - adaptive (also: close to "behavioral")
  - simple and efficient: computation, time, information
THERE EXIST *general, natural dynamics* leading to **CORRELATED EQUILIBRIA**

- "*general*": in all games
- "*natural*":
  - adaptive (also: close to "behavioral")
  - simple and efficient: computation, time, information
- "leading to correlated equilibria": statistics of play become close to **CORRELATED EQUILIBRIA**
The Involvement of the Orbitofrontal Cortex in the Experience of Regret

Nathalie Camille,1* Giorgio Coricelli,1,2* Jerome Sallet,1 Pascale Pradat-Diehl,3 Jean-René Duhamel,1 Angela Sirigu1†

Facing the consequence of a decision we made can trigger emotions like satisfaction, relief, or regret, which reflect our assessment of what was gained as compared to what would have been gained by making a different decision. These emotions are mediated by a cognitive process known as counterfactual thinking. By manipulating a simple gambling task, we characterized a subject’s choices in terms of their anticipated and actual emotional impact. Normal subjects reported emotional responses consistent with counterfactual thinking; they chose to minimize future regret and learned from their emotional experience. Patients with orbitofrontal cortical lesions, however, did not report regret or anticipate negative consequences of their choices. The orbitofrontal cortex has a fundamental role in mediating the experience of regret.
Brain, emotion and decision making: the paradigmatic example of regret

Giorgio Coricelli\textsuperscript{1}, Raymond J. Dolan\textsuperscript{2} and Angela Sirigu\textsuperscript{1}

\textsuperscript{1}Neuropsychology Group, Institut des Sciences Cognitives, CNRS, 67 Boulevard Pinel, 69675 Bron, France
\textsuperscript{2}Wellcome Department of Imaging Neuroscience, 12 Queen Square, London, WC1N 3BG, UK

Human decisions cannot be explained solely by rational imperatives but are strongly influenced by emotion. Theoretical and behavioral studies provide a sound empirical basis to the impact of the emotion of regret in guiding choice behavior. Recent neuropsychological and neuroimaging data have stressed the fundamental role of the orbitofrontal cortex in mediating the experience of regret. Functional magnetic resonance imaging data indicate that reactivation of activity within the orbitofrontal cortex and amygdala occurring during the phase of choice, when the brain is anticipating possible future consequences of decisions, characterizes the anticipation of regret. In turn, these patterns reflect learning based on cumulative emotional experience. Moreover, affective consequences can induce specific mechanisms of cognitive control of the choice processes, involving reinforcement or avoidance of the experienced behavior.

change. People, including those with a deep knowledge of optimal strategies, such as Markowitz, often try to avoid the likelihood of future regret, even when this conflicts with the prescription of decisions based on rational choice; according to the latter, individuals faced with a decision between multiple alternatives under uncertainty will opt for the course of action with maximum expected utility, a function of both the probability and the magnitude of the expected payoff [4].

Here, we outline, for the first time, the neural basis of the emotion of regret, and its fundamental role in adaptive behavior. The following questions will be addressed: what are the neural underpinnings of ‘powerful’ cognitively generated emotions such as regret? What are the theoretical implications of incorporating regret into the process of choice, and into adaptive models of decision making? In line with recent work on emotion-based decision making [5,6], we attempt to characterize the brain areas underlying
Decentralized Dynamic Spectrum Access for Cognitive Radios: Cooperative Design of a Non-Cooperative Game

Michael Maskery, Vikram Krishnamurthy, and Qing Zhao

Abstract—We consider dynamic spectrum access among cognitive radios from an adaptive, game theoretic learning perspective. Spectrum-agile cognitive radios compete for channels temporarily vacated by licensed primary users in order to satisfy their own demands while minimizing interference. For both slowly varying primary user activity and slowly varying statistics of “fast” primary user activity, we apply an adaptive regret based learning procedure which tracks the set of correlated equilibria of the game, treated as a distributed stochastic approximation. This procedure is shown to perform very well compared with other similar adaptive algorithms. We also estimate channel contention for a simple CSMA channel sharing scheme.

Index Terms—Cognitive radio, dynamic spectrum access, game theory, stochastic approximation, correlated equilibrium.
Fig. 6. Long-run average spectrum utilization in a dynamic environment for the channel allocation techniques of Section V. $T(\rho)$ (see Sec.VI-C) is the mean time between innovations in the system (changes in primary user activity, fast primary user statistics or cognitive radio demands).
Fully non-cooperative optimal placement of mobile vehicles

Shemin Kalam, Mahbub Gani and Lakmal Seneviratne

Abstract— In this paper, we consider optimal placement of autonomous mobile vehicles such that a cost function involving all the vehicles and possible locations of targets is minimized. This cost is proportional to the distance between the targets and vehicles. The optimal locations correspond to the vehicles being at the centroids of their own Voronoi cell which correspond to Centroidal Voronoi Tessellations (CVTs). We have adopted a game theoretical formulation to initially consider vehicle target assignment where a set of mobile vehicles choose their own targets. The movement of the vehicles towards the optimal locations is based on MacQueen’s algorithm. But an important step of MacQueen’s algorithm requires the knowledge of the nearest neighbour to be determined from a sample that is drawn from a fixed but unknown probability distribution. This calculation seems to be implicit in reported algorithms and brings in a hidden centralized process. We have used game theory as a framework to get around this problem and modelled the vehicles such that they are capable of making their own decisions and interested in optimizing their own utilities. Specifically, we have introduced an appropriate utility function and require the vehicles to negotiate their choice of targets via regret matching. We present simulations that illustrate that vehicles choose the targets optimally and converge to CVTs.

Lloyd’s descent algorithm [15] can be applied to solve the problem and it has been shown by Cortés et al [12] that the distribution of mobile sensors converges to Centroidal Voronoi Tessellations (CVTs). Considering a scenario where the spatial distribution is not known, MacQueen’s algorithm [16] is a Monte-Carlo method of solving the problem [22]; in our problem we interpret MacQueen’s algorithm as a real-time higher order control strategy. A drawback of the MacQueen’s algorithm is that it requires the calculation of nearest neighbours. We have drawn upon game theory to get around the problem of this requirement. This reduces the communication burden on the system because the vehicles do not require the information about the distances between all the vehicles and targets which would have been needed for calculation of the nearest neighbour.

The game theoretic strategy that we adopt is inspired by the problem of autonomous vehicle-target assignment tackled by Arslan in [8]. To get to the problem of optimal placement of the mobile vehicles, initially we consider how a group of vehicle are to optimally assign themselves to a set of targets.
Network-Enabled Missile Deflection: Games and Correlation Equilibrium

MICHAEL MASKERY
VIKRAM KRISHNAMURTHY
University of British Columbia

The problem of deploying countermeasures (CM) against antiship missiles is investigated from a network centric perspective in which multiple ships coordinate to defend against a known missile threat. Using the paradigm of network enabled operations (NEOPS), the problem is formulated as a transient stochastic game with communication where the appropriate strategy takes the form of an optimal stationary correlated equilibrium. Under this strategy, ships cooperate through real-time communication to satisfy both local and collective interests. The use of communication results in a performance improvement over the noncommunicating, Nash equilibrium scenario. This framework allows us to develop a theoretical foundation for NEOPS and captures the trade-off between information exchange and performance, while generalizing the standard Nash equilibrium solution for the missile deflection game given in [1]. The NEOPS equilibrium strategy is characterized as the solution to an optimization problem with linear objective and bilinear constraints, which can be solved calculating successive improvements starting from an initial noncooperative (Nash) solution. The communication overhead required to implement this strategy is associated with the mutual information between individual action probability distributions at equilibrium. Numerical results illustrate the trade-off between communication and performance.
Predicting Human Interactive Learning by Regret-Driven Neural Networks

Davide Marchiori$^1$ and Massimo Warglien$^2$*

Much of human learning in a social context has an interactive nature: What an individual learns is affected by what other individuals are learning at the same time. Games represent a widely accepted paradigm for representing interactive decision-making. We explored the potential value of neural networks for modeling and predicting human interactive learning in repeated games. We found that even very simple learning networks, driven by regret-based feedback, accurately predict observed human behavior in different experiments on 21 games with unique equilibria in mixed strategies. Introducing regret in the feedback dramatically improved the performance of the neural network. We show that regret-based models provide better predictions of learning than established economic models.
Classification of peptide mass fingerprint data by novel no-regret boosting method

Anna Gambin\textsuperscript{a,*}, Ewa Szczurek\textsuperscript{a,b}, Janusz Dutkowski\textsuperscript{a}, Magda Bakun\textsuperscript{c}, Michał Dadlez\textsuperscript{c,d}

\textsuperscript{a}Institute of Informatics, Warsaw University, Banacha 2, 02-097 Warsaw, Poland
\textsuperscript{b}Max Planck Institute for Molecular Genetics, Ihnestasse 73, 14195 Berlin, Germany
\textsuperscript{c}Institute of Biochemistry and Biophysics PAS, Pawińskiego 5A, 02-106 Warsaw, Poland
\textsuperscript{d}Biology Department, Warsaw University, Miecznikowa 1, 02-096 Warsaw, Poland

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FTICR
LC-MS
Boosting
Classifier
Cystic fibrosis
Ovarian cancer

\textbf{ABSTRACT}

We have developed an integrated tool for statistical analysis of large-scale LC-MS profiles of complex protein mixtures comprising a set of procedures for data processing, selection of biomarkers used in early diagnostic and classification of patients based on their peptide mass fingerprints.

Here, a novel boosting technique is proposed, which is embedded in our framework for MS data analysis. Our boosting scheme is based on Hannan-consistent game playing strategies. We analyze boosting from a game-theoretic perspective and define a new class of boosting algorithms called H-boosting methods. In the experimental part of this work we apply the new classifier together with classical and state-of-the-art algorithms to classify ovarian cancer and cystic fibrosis patients based on peptide mass spectra.

The methods developed here provide automatic, general, and efficient means for processing of large scale LC-MS datasets. Good classification results suggest that our approach is able to uncover valuable information to support medical diagnosis.
NASH EQUILIBRIUM: a fixed-point of a non-linear map
Dynamics and Equilibrium

- **Nash equilibrium**: a *fixed-point* of a non-linear map

- **Correlated equilibrium**: a solution of finitely many *linear inequalities*
"LAW OF CONSERVATION OF COORDINATION":
"LAW OF CONSERVATION OF COORDINATION":

There must be some COORDINATION —
"Law of Conservation of Coordination":

There must be some coordination —
either in the equilibrium notion,
"Law of Conservation of Coordination":

There must be some Coordination — either in the Equilibrium notion, or in the Dynamic
The "Program"
The "Program"

A. **Demarcate** the **BORDER** between
The "Program"

A. **Demarcate** the **BORDER** between classes of dynamics where convergence to equilibria **CAN** be obtained
The "Program"

A. **Demarcate** the **BORDER** between

- classes of dynamics where convergence to equilibria **CAN** be obtained, and

- classes of dynamics where convergence to equilibria **CANNOT** be obtained
A. **Demarcate** the **BORDER** between

- classes of dynamics where convergence to equilibria **CAN** be obtained, and

- classes of dynamics where convergence to equilibria **CANNOT** be obtained

B. **Find** **NATURAL** dynamics for the various equilibrium concepts
Dynamics and Equilibrium
Dynamics and Equilibrium
Dynamics and Equilibrium