



Yisrael Aumann's Science

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Israel Academy of Sciences and Humanities

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Aumann's Short CV

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- *1998-2003:* Founding President of the Game Theory Society

2005



Major Contributions

- **Repeated games**

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- Perfect competition

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- Correlated equilibrium

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- **Interactive epistemology**

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- **Foundations**
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Repeated Games

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Aumann 1981, 1985

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“The theory of *repeated games* ... Its aim is to account for phenomena such as **cooperation, altruism, revenge, threats** (self-destructive or otherwise) ... **concealing and revealing information** ... etc.—phenomena which may at first seem irrational—in terms of the usual ‘**selfish**’ **utility-maximizing paradigm** of game theory and neoclassical economics.”

Aumann 1981, 1985

Repeated Games

**Noncooperative strategic behavior
in the repeated game**

yields

Cooperative behavior

Repeated Games

- **The Folk Theorem**

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- **The Strong Folk Theorem**
(*Aumann 1959*)
- **The Perfect Folk Theorem**
(*Aumann & Shapley 1976, Rubinstein 1976*)
- **Repeated Games
of Incomplete Information**
(*Aumann & Maschler 1966,
Aumann, Maschler & Stearns 1968*)

The Market



Pieter Bruegel the Elder (1559)

The Market Clears



Ursus Wehrli, *Tidying Up Art* (2002)

Perfect Competition

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Aumann 1964

Perfect Competition

How should **perfect competition** be modelled?

“... the influence of an individual participant on the economy *cannot be* mathematically *negligible*, as long as there are *only finitely many* participants.

... We submit that the most natural model for this purpose contains a **continuum** of participants, similar to the continuum of points on a line or the continuum of particles in a fluid.”

Aumann 1964

The Equivalence Principle

In markets with a continuum of traders:

The Equivalence Principle

In **markets with a continuum of traders:**

The set of ***Walrasian equilibria***

coincides with

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*Aumann 1964, Aumann & Shapley 1974,
Aumann 1975, ...*

Correlated Equilibrium

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A **Correlated Equilibrium** is a Nash equilibrium when the players receive payoff-irrelevant **signals** before playing the game

Aumann 1974

Interactive Epistemology

- **Formal model** of knowledge, knowledge about knowledge, and common knowledge

Aumann 1976, 1999ab

Interactive Epistemology

- **Formal model** of knowledge, knowledge about knowledge, and common knowledge

- **The Agreement Theorem:**

*If two people have the same prior,
and their posteriors for an event A are
common knowledge,
then their posteriors **must be equal**.*

Aumann 1976, 1999ab

Rationality

Assume a common prior.

Aumann 1987

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If all players are **rational**,*

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Rationality

Assume a common prior.

*If all players are **rational**,
and this is **common knowledge**,*

Aumann 1987

Rationality

Assume a common prior.

*If all players are **rational**,
and this is **common knowledge**,*

then

*their play constitutes
a **correlated equilibrium***

Aumann 1987

Other Major Contributions

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The Unified Game Theory

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“Unlike other approaches to disciplines like economics or political science, **GAME THEORY** **does not use different, ad-hoc constructs** to deal with various specific issues, such as perfect competition, monopoly, oligopoly, international trade, taxation, voting, deterrence, animal behavior, and so on.

Aumann's interview 2004

The Unified Game Theory

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Rather, it develops methodologies that apply in principle to all interactive situations, then sees where these methodologies lead in each specific application.”

Aumann's interview 2004

Aumann's Doctoral Students

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1. Bezalel Peleg
2. David Schmeidler
3. Shmuel Zamir
4. Elon Kohlberg
5. Benyamin Shitovitz
6. Zvi Artstein
7. Eugene Wesley

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8. Sergiu Hart
9. Abraham Neyman
10. Yair Tauman
11. Dov Samet
12. Ehud Lehrer
13. Yossi Feinberg
14. Itai Arieli

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Aumann

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S. Hart, Introduction to Aumann's interview 2004

Aumann

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He has shaped the field through his fundamental and pioneering work, work that is conceptually profound, and much of it also mathematically deep.”

S. Hart, Introduction to Aumann's interview 2004

A Scientist at Play



A Scientist at Work

