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A COMPARISON OF NON-TRANSFERABLE UTILITY VALUES*

ABSTRACT. Three values for non-transferable utility games – the Harsanyi NTU-value, the Shapley NTU-value, and the Maschler–Owen consistent NTU-value – are compared in a simple example.

KEY WORDS: cooperative games, non-transferable utility (NTU), value, NTU-value, Shapley value, Harsanyi value, Maschler–Owen value, consistent value

1. INTRODUCTION

A general *non-transferable utility game in coalitional form* – an *NTU-game* for short – is given by its set of players and the sets of outcomes that are feasible for each subset ("coalition") of players.

A central solution concept for coalitional games is that of *value*, originally introduced by Shapley (1953) for games with *transferable utility* (or *TU-games* for short). For *pure bargaining problems* – where only the grand coalition of all players is essential – the classical solution is the Nash (1950) *bargaining solution*. Since the TU-games and the pure bargaining problems are two special classes of NTU-games, one looks for an NTU-solution that extends both.

An *NTU-value* is thus a solution concept for NTU-games that satisfies the following: first, it coincides with the Shapley TU-value for TU-games; second, it coincides with the Nash bargaining solution for pure bargaining problems; and third, it is covariant with individual payoff rescalings (i.e., multiplying

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the payoffs of a player by a factor $\alpha > 0$ multiplies his value payoffs by the same factor α).

The above three requirements do not however determine the NTU-value uniquely. Indeed, different NTU-values have been proposed in the literature; the most notable are due to Harsanyi (1963), Shapley (1969), and Maschler and Owen (1992).[1]

In this short note we analyze in detail a simple example of an NTU-game where the three values yield different outcomes. It is essentially the simplest possible example: there are just three players (a two-player game is a pure bargaining problem), and the coalitional function corresponds to a TU-game except for one coalition, for which the transfers of utility are possible albeit at a rate different from 1. The difference between the values will be seen to be due to the way that subcoalitions are handled.[2] It is to be hoped that the analysis here will shed further light on the NTU-values, their meanings and interpretations.

The reader is referred to the chapters on value in the *Handbook of Game Theory*, in particular McLean (2002), for further material and references.

The game is defined in Section 2; the values are computed in Sections 3–5, and compared in Section 6. Sections 7 and 8 present an exchange economy (a ‘‘market’’) and a ‘‘prize game’’ (see Hart (1994)) that generate our example.

Some notations: $\mathbb{R}$ is the real line; for a finite set S, the number of elements of S is denoted $|S|$; the $|S|$-dimensional Euclidean space with coordinates indexed by S (or, equivalently, the set of real functions on S) is $\mathbb{R}^S$; the nonnegative orthant of $\mathbb{R}^S$ is $\mathbb{R}^S_+$; and $A \subset B$ denotes *weak* inclusion (i.e., possibly $A = B$).

2. THE EXAMPLE

A *non-transferable utility game in coalitional form* is a pair (N, V), where N – the set of *players* – is a finite set, and V – the *coalitional function* – is a mapping that associates to each coalition $S \subset N$ the set $V(S) \subset \mathbb{R}^S$ of *feasible payoff vectors* for S. An element $x = (x^i)_{i \in S}$ of $V(S)$ is interpreted as follows: there exists an outcome that is feasible for the coalition S whose utility to player i is x^i (for each i in S). Thus $V(S)$ is the set of utility combinations that are feasible for the coalition S. The standard assumptions are that for each nonempty coalition S, the set $V(S)$ is a nonempty strict subset of $\mathbb{R}^S$ that is closed, convex, and comprehensive ($y \leq x \in V(S)$ implies $y \in V(S)$).

A game (N, V) is a *transferable utility game* (or *TU-game* for short) if for each coalition S there exists a real number $v(S)$ such that $V(S) = \{x \in \mathbb{R}^S : \sum_{i \in S} x^i \leq v(S)\}$. This game is denoted (N, V) or (N, v) interchangeably, and the function v is called the *worth* function.

Our example is the NTU-game (N, V) with three players $N = \{1, 2, 3\}$ and coalitional function[3]

$$\begin{aligned} V(i) &= \{x^i : x^i \leq 0\} \quad \text{for } i = 1, 2, 3, \\ V(12) &= \{(x^1, x^2) : x^1 + x^2 \leq 36,\ x^1 + 2x^2 \leq 36\}, \\ V(13) &= \{(x^1, x^3) : x^1 + x^3 \leq 0\}, \\ V(23) &= \{(x^2, x^3) : x^2 + x^3 \leq 0\}, \\ V(123) &= \{(x^1, x^2, x^3) : x^1 + x^2 + x^3 \leq 36\}. \end{aligned}$$

Except for coalition $\{1, 2\}$ – whose feasible set $V(12)$ is depicted in Figure 1 – our game (N, V) coincides with a TU-game, which we denote by (N, w) or (N, W). Thus w is the worth function

$$w(S) = \begin{cases} 36, & \text{for } S = \{1, 2\}, \{1, 2, 3\}, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

and $W(S) = \{x \in \mathbb{R}^S : \sum_{i \in S} x^i \leq w(S)\}$ for all S.

The game (N, V) is 0-*normalized* (single players get 0) and *monotonic* (if $S \subset T$ and $x \in V(S)$ then[4] $(x, 0^{T \setminus S}) \in V(T)$). The (Pareto efficient) boundary of $V(N)$, which is denoted $\partial V(N)$, is a hyperplane with slope $\lambda = (1, 1, 1)$.

This example is not new; a similar one appears in Owen (1972) (see also Hart and Mas-Colell (1996, p. 369)).

3. THE SHAPLEY NTU-VALUE

The Shapley NTU-value for a general NTU-game (N, V) is obtained by the following procedure:

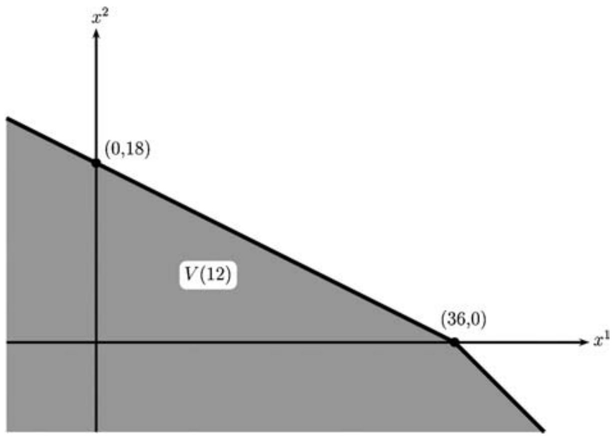

Figure 1. The feasible set for coalition {1, 2}.

- For each weight vector $\lambda \in \mathbb{R}^N_+, \lambda \neq 0$:

  1. Let the payoff vector $z \in \mathbb{R}^N$ satisfy

     $\lambda^i z^i = \varphi^i_{\mathrm{TU}}(N, v_\lambda) \quad \text{for all } i \in N,$

     where the TU-game (N, v_λ) is obtained from (N, V) by allowing transfers of utilities at the rates λ, i.e., $v_\lambda(S) = \sup\{\sum_{i\in S} \lambda^i x^i : (x^i)_{i\in S} \in V(S)\}$ for all[5] $S \subset N$, and φ_{TU} is the *Shapley TU-value*.
  2. If z is feasible for the grand coalition, i.e., if $z \in V(N)$, then z is a Shapley NTU-value of (N, V).

For our game (N, V) only $\lambda = (1, 1, 1)$ needs to be considered (any λ that is not a multiple of $(1, 1, 1)$ yields $v_\lambda(N) = \infty$), in which case $v_\lambda \equiv w$ (see (1)). The Shapley TU-value σ_S of each subgame[6] (S, w) is easily computed: $\sigma_{\{i\}} = 0$ for singletons, and $\sigma_{\{1,2\}} = (18, 18)$, $\sigma_{\{1,3\}} = (0, 0)$, and $\sigma_{\{2,3\}} = (0, 0)$ for the two-player subgames. It will be convenient to write σ_S as a three-dimensional vector with "–" for the players outside S:

$$\sigma_{\{1,2\}} = (18, 18, -),$$
$$\sigma_{\{1,3\}} = (0, -, 0),$$
$$\sigma_{\{2,3\}} = (-, 0, 0).$$

Next, for each two-player coalition S we adjoin to σ_S a payoff for the missing player (in boldface below) so that the resulting payoff vector $\hat{\sigma}_S$ is efficient for N (i.e., the coordinates add up to 36):

$$\hat{\sigma}_{\{1,2\}} = (18, 18, \mathbf{0}),$$
$$\hat{\sigma}_{\{1,3\}} = (0, \mathbf{36}, 0),$$
$$\hat{\sigma}_{\{2,3\}} = (\mathbf{36}, 0, 0).$$

We then average these three vectors to obtain the value for N:

$$\sigma_N = (18, 18, 0).$$

Indeed, the Shapley TU-value of a player i in a TU-game (N, v) is the average of his marginal contribution to the grand coalition $v(N) - v(N\backslash i)$, and his values in the subgames with $|N| - 1$ players:

$$\varphi^i_{\mathrm{TU}}(N, v) = \frac{1}{|N|}\left[v(N) - v(N\backslash i) + \sum_{j \in N\backslash i} \varphi^i_{\mathrm{TU}}(N\backslash j, v)\right]; \qquad (2)$$

see Hart and Mas-Colell (1996, p. 369).

The above payoff vector σ_N is thus the unique Shapley NTU-value of our game (N, V).

4. THE HARSANYI NTU-VALUE

The Harsanyi NTU-value for a general NTU-game (N, V) is obtained by the following procedure:

- For each weight vector $\lambda \in \mathbb{R}^N_+, \lambda \neq 0$:
  1. Let the payoff vector $z \in \partial V(N)$ be the *λ-egalitarian solution* of the game (N, V).
  2. If λ is a supporting normal to the boundary of $V(N)$ at z (i.e., if z is also *λ-utilitarian*) then z is a *Harsanyi NTU-value* of (N, V).

The λ-egalitarian solution is constructed recursively: for each S, given the payoff vectors $\eta_T \in V(T)$ for all strict subsets T of S, the payoff vector η_S is determined by

$$\eta_S \in \partial V(S) \quad \text{and}$$
$$\lambda^i(\eta^i_S - \eta^i_{S\backslash j}) = \lambda^j(\eta^j_S - \eta^j_{S\backslash i}) \quad \text{for all } i, j \in S. \qquad (3)$$

The λ-egalitarian solution z is the resulting payoff vector η_N for the grand coalition.

For our game (N, V) we need to consider only $\lambda = (1, 1, 1)$. This yields $\eta_{\{i\}} = 0$ for all i, and

$$\eta_{\{1,2\}} = (12, 12, -),$$

$$\eta_{\{1,3\}} = (0, -, 0),$$

$$\eta_{\{2,3\}} = (-, 0, 0).$$

Indeed, for each two-player coalition $\{i, j\}$ the egalitarian solution is the payoff vector $x \in \partial V(ij)$ with equal coordinates (i.e., $x^i = x^j$).

For the grand coalition N we use the same construction as for the Shapley TU-value. First, we extend each η_S to a payoff vector $\hat{\eta}_S$ that is efficient for N:

$$\hat{\eta}_{\{1,2\}} = (12, 12, \mathbf{12}),$$

$$\hat{\eta}_{\{1,3\}} = (0, \mathbf{36}, 0),$$

$$\hat{\eta}_{\{2,3\}} = (\mathbf{36}, 0, 0);$$

and then we average the three vectors to yield η_N:

$$\eta_N = (16, 16, 4).$$

This construction is correct since the egalitarian solution of (N, V) is the Shapley TU-value of the game (N, u) with $u(S) = \Sigma_{i \in S} \eta_S^i$ for all $S \subset N$; see Hart (1985, (4.6)).

Thus η_N is the egalitarian solution for N, and therefore the unique Harsanyi NTU-value of (N, V); in terms of Hart (1985), the collection $(\eta_S)_{S \subset N}$ we have obtained is the Harsanyi *payoff configuration*.

5. THE MASCHLER–OWEN CONSISTENT NTU-VALUE

The Maschler–Owen consistent NTU-value for a general NTU-game (N, V) is obtained recursively by the following procedure:

- Let S be a coalition, and assume that a payoff vector $\gamma_T \in V(T)$ is given for all strict subcoalitions T of S. For each weight vector $\lambda \in \mathbb{R}_+^S, \lambda \neq 0$:

1. Let the payoff vector $z \in \mathbb{R}^S$ satisfy

$$\lambda^i z^i = \frac{1}{|S|}\left[v_\lambda(S) - \sum_{j \in S \setminus i} \lambda^j \gamma^j_{S \setminus i} + \sum_{j \in S \setminus i} \lambda^i \gamma^i_{S \setminus j} \right] \quad \text{for all } i \in S, \tag{4}$$

where, again, $v_\lambda(S) = \sup\{\sum_{i \in S} \lambda^i x^i : (x^i)_{i \in S} \in V(S)\}$.

2. If z is feasible for the coalition S, i.e., if $z \in V(S)$, then define $\gamma_S = z$.

- The resulting payoff vector γ_N for the grand coalition is then a *Maschler–Owen consistent NTU-value* of (N, V).

Formula (4) – which is a generalization of (2) in the TU-case – is equivalent to[7]

$$\sum_{i \in S} \lambda^i z^i = v_\lambda(S) \quad \text{and}$$
$$\sum_{j \in S \setminus i} \lambda^i (z^i - \gamma^i_{S \setminus j}) = \sum_{j \in S \setminus i} \lambda^j (z^j - \gamma^j_{S \setminus i}) \quad \text{for all } i \in S;$$

see Proposition 4 and Formula (3) in Hart and Mas-Colell (1996), and Formula (5.1) in Maschler and Owen (1989).

For our game (N, V) we have $\gamma_{\{i\}} = 0$ for all i, and

$$\gamma_{\{1,2\}} = (18, 9, -),$$
$$\gamma_{\{1,3\}} = (0, -, 0),$$
$$\gamma_{\{2,3\}} = (-, 0, 0).$$

Indeed, for two players, the consistent value coincides with the Nash bargaining solution.

Now (4) allows us to use again the "extension" construction ($\lambda = (1, 1, 1)$):

$$\hat{\gamma}_{\{1,2\}} = (18, 9, \mathbf{9}),$$
$$\hat{\gamma}_{\{1,3\}} = (0, \mathbf{36}, 0),$$
$$\hat{\gamma}_{\{2,3\}} = (\mathbf{36}, 0, 0),$$

and their average is

$$\gamma_N = (18, 15, 3).$$

Therefore γ_N is the unique Maschler–Owen consistent NTU-value of (N, V).

6. A COMPARISON

To recapitulate, the three NTU-values of our game (N, V) are

$$\begin{aligned} \varphi^{\mathrm{Sh}}(N, V) &= (18, 18, 0), \\ \varphi^{\mathrm{Ha}}(N, V) &= (16, 16, 4), \\ \varphi^{\mathrm{MO}}(N, V) &= (18, 15, 3). \end{aligned} \tag{5}$$

The computations leading to these values clearly exhibit that the difference between them derives from the way the intermediate payoff vector $x_{\{1,2\}}$ for the coalition $\{1, 2\}$ – the only coalition whose feasible set is *not* of the TU-type – is determined. Indeed, the payoff vectors x_S for all other strict subsets of N are identical for the three values, and, once all the x_S are given, the value for the grand coalition x_N is uniquely determined (by the "extension" construction with respect to $\lambda = (1, 1, 1)$, the unique supporting normal to $\partial V(N)$).

The payoff vectors $x_{\{1,2\}}$ for the coalition $\{1, 2\}$ are, respectively,

$$\begin{aligned} x^{\mathrm{Sh}}_{\{1,2\}} &= (18, 18), \\ x^{\mathrm{Ha}}_{\{1,2\}} &= (12, 12), \\ x^{\mathrm{MO}}_{\{1,2\}} &= (18, 9); \end{aligned}$$

(see Figure 2). The Shapley NTU-value and the Harsanyi NTU-value both take $x_{\{1,2\}}$ to be an egalitarian outcome (i.e., an "equal-split" payoff vector – since the rates of interpersonal utility comparison λ dictated by the grand coalition satisfy $\lambda^1 = \lambda^2$). The difference is that the Harsanyi approach uses $V(12)$, the feasible set for $\{1, 2\}$, to determine $x_{\{1,2\}}$, whereas the Shapley approach allows transfers of utility at the rates λ of the grand coalition and so $V(12)$ is replaced by $W(12)$ (which corresponds to $v_\lambda(12)$). Thus $x^{\mathrm{Ha}}_{\{1,2\}} = (12, 12) \in \partial V(12)$ and $x^{\mathrm{Sh}}_{\{1,2\}} = (18, 18) \in \partial W(12)$. As for the Maschler–Owen NTU-value, it considers coalition $\{1, 2\}$ independently of the grand

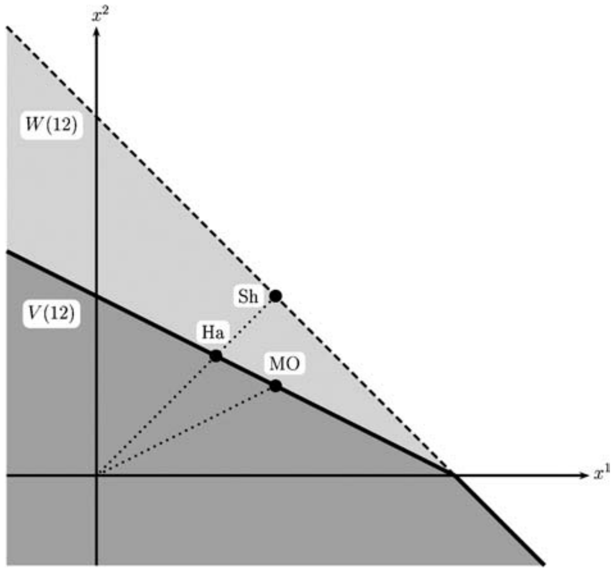

Figure 2. The intermediate payoff vectors for coalition {1, 2}.

coalition: $x_{\{1,2\}}$ is determined by the $\{1,2\}$-subgame only. Moreover, $x_{\{1,2\}}$ is determined for $\{1,2\}$ in exactly the same way that x_N is determined for *N*; this property – that x_S is the consistent NTU-value of the *S*-subgame for each *S* – is called "subcoalition perfectness" in Hart and Mas-Colell (1996, p. 366). Thus $x^{\mathrm{MO}}_{\{1,2\}} = (18, 9)$, the Nash bargaining solution of the two-person game.

Which approach is "correct"? There cannot be a definite answer.[8] For instance, it may depend on the way the interactions between the players are conducted.[9] If transfers are allowed (or "implied" by the grand coalition[10]) as in the Shapley NTU-value, then player 3 becomes a null ("dummy") player, and his value of 0 is justified. Otherwise player 3 is not a null player, and his value is positive. The Harsanyi NTU-value is egalitarian-based; therefore players 1 and 2 get equal payoffs. In contrast, the Maschler–Owen NTU-value takes into account the asymmetry between the two players in the subcoalition $\{1,2\}$ – and is the only one to do so. Thus it appears that the Maschler–Owen consistent NTU-value reflects the structure of this game better than the other NTU-values.[11]

7. AN EXCHANGE ECONOMY

Our example is essentially a market game.[12] For instance, take E to be the following exchange economy ("market"): there are three players ("traders") $i = 1, 2, 3$, and three commodities; the utility functions are

$$u^1(a_1, a_2, a_3) = 36a_1 + 36a_2 - 36,$$

$$u^2(a_1, a_2, a_3) = 18a_1 + 36a_3,$$

$$u^3(a_1, a_2, a_3) = 36a_1 + 36a_3 - 36$$

(a_j denotes the quantity of good j), and the initial commodity bundles ("endowments") are

$$e^1 = (1, 0, 0),$$

$$e^2 = (0, 1, 0),$$

$$e^3 = (0, 0, 1).$$

Let (N, V^E) be the resulting *NTU-market game*; i.e., $V^E(S) = \{x \in \mathbb{R}^S$: there exists an S-allocation $(c^i)_{i \in S}$ with $\Sigma_{i \in S}\, c^i = \Sigma_{i \in S}\, e^i, c^i \in \mathbb{R}^3_+$ and $x^i \leq u^i(c^i)$ for all $i \in S\}$ for all $S \subset N = \{1, 2, 3\}$. The individually rational payoff vectors of (N, V^E) coincide with those of our example (N, V); i.e., for all $S \subset N$ we have $V^E(S) \cap \mathbb{R}^S_+ = V(S) \cap \mathbb{R}^S_+$ (note that $u^i(e^i) = 0$ and $\partial V(i) = \{0\}$ for all i), and also $V^E(S) \subset V(S)$. This implies that the NTU-values of (N, V) given in (5) are also NTU-values for (N, V^E). One can check that (N, V^E) has no other values.[13]

8. A PRIZE GAME

Our example is also essentially a *hyperplane game*, and it can thus be represented as a *prize game*; see Hart (1994). Indeed, let the prize of the grand coalition be worth 36 to each player, and let the prize of coalition $\{1, 2\}$ be worth 36 to player 1 and 18 to player 2 (there are no other prizes). The resulting game (N, V^*) is again identical to our example (N, V) in the individually rational region, and its NTU-values are given by (5).

NOTES

1. The Shapley NTU-value is sometimes referred to as the 'λ-transfer value,' and the Maschler–Owen value is called the 'consistent NTU-value.' Axiomatizations of these values have been provided by Aumann (1985) for the Shapley NTU-value, by Hart (1985) for the Harsanyi NTU-value, by de Clippel, Peters and Zank (2002) and Hart (1994, 2003) for the Maschler–Owen NTU-value. Another NTU-value was proposed by Owen (1972).
2. See also the discussions in Hart (1985, Section 5), Hart and Mas-Colell (1996, Section 4), and de Clippel et al. (2002, Section 4).
3. For simplicity we write $V(1)$, $V(12), \ldots$ instead of the more cumbersome $V(\{1\})$, $V(\{1,2\})$, ...; similarly, $S\backslash i$ for $S\backslash\{i\}$, and so on.
4. $0^{T\backslash S}$ is the 0-vector in $\mathbb{R}^{T\backslash S}$.
5. If the 'sup' in the definition $v_\lambda(S)$ is infinite for some S then there is no NTU-value corresponding to this λ (and the procedure for this λ stops here).
6. The 'subgame' (S, V) of (N, V) is obtained by restricting the domain of V to the subsets of S.
7. Compare (3).
8. For a general discussion of the multiplicity of solution concepts, see '1930–1950, Section iii' in Aumann (1987).
9. Different NTU-values may be thought of, inter alia, as corresponding to different bargaining procedures – from which the coalitional form abstracts away. For example, the noncooperative model of Hart and Mas-Colell (1996) leads to the Maschler–Owen NTU-value (and thus, a fortiori, to the Shapley TU-value and the Nash bargaining solution). It would be of interest to obtain explicit bargaining procedures leading to other NTU-values.
10. See Myerson (1991, pp. 475–476).
11. To emphasize the differences between the values, consider the case where $V(12)$ becomes more and more "flat": replace $x^1 + 2x^2 \leq 36$ with $x^1 + mx^2 \leq 36$, and let $m \to \infty$. Then $\varphi^{\text{Sh}} = (18, 18, 0)$, $\varphi^{\text{Ha}} \to (12, 12, 12)$, and $\varphi^{\text{MO}} \to (18, 12, 6)$.
12. I.e., it coincides with a market game in the relevant (individually rational) region; see below.
13. Since $\lambda = (1, 1, 1)$ is no longer the unique supporting normal to the boundary of $V^E(N)$, one needs to consider additional weight vectors λ (including zero weights). We omit the straightforward but lengthy arguments that show that no other values are obtained.

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