THE GAME OF POLITICS

By ROBERT J. AUMANN

Anatol Rapoport, *Fights, Games, and Debates*, Ann Arbor, The University of Michigan Press, 1960, 400 pp. $6.95.

THE last decade has seen a tremendous outpouring of work on the applications of mathematics to the non-exact sciences. In the accepted jargon, this activity is called "model-building"; mathematical models have been built in disciplines ranging from biology and psychology to economics, sociology, linguistics, and even politics. One of the most engaging of these models is the whimsically named "theory of games." More to the point would have been to call it a "theory of conflict and cooperation"; but mathematicians like to christen their babies according to their appearance, not according to the job they are expected to do. Mathematically, wars and games look alike,[1] and if one wants to build a model for bargaining, then it doesn't matter whether the subject is disarmament or coffee breaks.

Professor Rapoport has really written not one but three books on three utterly different approaches to the problems of conflict and cooperation. The "fights" part of his book deals with "social physics"—i.e., with mathematical models for mass action, action on which the individual has no influence. The "games" part describes game theory—the mathematical model based on individual rationality. The "debates" part constitutes a psychological analysis of the art of winning over other people to your point of view; its main burden is that in order to win them over, you first have to get them to *listen* to you. Unlike the first two, this third part uses no mathematical tools. The three parts of the book all deal with conflict and cooperation in one form or another, but otherwise they have nothing in common; wisely, the author has not attempted to establish any spurious connections. This reviewer feels competent and willing to discuss only the "games" part of the book.

This constitutes by far the best existing popularization of game theory.[2] It is lucid, informative, and exciting; it offers significant in-

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[1] They also look alike in the Bible: "Abner said to Joab, 'Let the young men . . . play before us.'. . . And they caught every one his fellow by the head, and thrust his sword in his fellow's side. . . . There was a very sore battle that day." (II Samuel 2: 14-17.)

[2] The book by R. Duncan Luce and Howard Raiffa (*Games and Decisions*, New York, 1957) is more accurate and complete, and, like the book before us, makes excellent reading. But it is not a popularization; it cannot be thoroughly understood by readers with no previous mathematical training.

sights into the philosophical background and consequences of the theory, and suggests interesting possible new departures. A feature of the book are the numerous examples taken from all walks of life and literature (quite a few from *War and Peace*). The author uses these examples to illuminate the ideas he introduces, and conversely these ideas throw new light on the examples. Urbane, witty, and sophisticated, his work is altogether a delight to read. For the reader who wants to know what the shouting is all about, it is excellent. This reviewer does have some serious reservations; but inaccuracies are inevitable in any non-technical exposition of a highly technical and sophisticated subject. On the whole, the book cannot be too highly recommended, and I warmly congratulate Professor Rapoport for being the first to make the basic ideas of game theory available to the world at large.

I

Social physics, the subject of the "fights" part of the book, is a *descriptive* theory; that is, the conclusions drawn from this theory are (or should be) subject to empirical verification. Nothing is assumed about how people ought to behave, or how they would behave if they were sensible; that is irrelevant. For example, one part of social physics concerns arms races; the equations that are developed are meant to describe the development of actual arms races, in the same sense that the equations of thermodynamics are meant to describe the actual behavior of gases. Thus the general aims of social physics are similar in spirit to those of natural science.

Not so with game theory; this is a *normative* theory. It gives advice, describes the behavior (in conflict-cooperation situations) of theoretical beings called "rational persons," suggests arbitration schemes, defines "fair outcomes," and so on. It does *not* purport to describe actual behavior of any kind, this being notoriously irrational. The general aims of game theory are comparable to those of certain parts of applied statistics, such as quality control, or to those of operations research.[3] It is more complicated than these because it must take conflicting interests into account, but like them it seeks to give good, "efficient" advice, notwithstanding the inefficient behavior that is usually observed.

Which approach is more suited to international politics? It depends on what your interest in international politics is. If you are a political scientist who is primarily interested in describing politics as practiced, then social physics or some other purely descriptive approach is the

[3] Decision theory is an inclusive name for all these disciplines (it includes game theory).

right one. On the other hand, if you are advising a policy-maker, or if you *are* a policy-maker,[4] the game-theoretic approach is the more appropriate. To be sure, even then social physics is not irrelevant, because it may give you some idea as to what to expect from groups other than yours. Basically, though, the policy-maker needs a normative rather than a descriptive theory—to the extent that he needs or can use theories at all.

II

Preferences are the basic entities with which any normative theory of conflict and cooperation must deal; if an individual doesn't know what he wants, not even a mathematician can help him. To deal with preferences efficiently, it is desirable to translate them into numerical terms. This is what is accomplished by the theory of *utility*. Chapter VI, the first chapter of the "games" part of Rapoport's book, is an exposition of utility theory. I must emphasize that the utility concept is a tool useful and, indeed, indispensable to the mathematician for proving his results, but no more than that; contrary to a prevalent misconception (which the book does not dispel), all the results of game theory that are usually stated in terms of utility can be stated in terms of preferences only, without ever mentioning the word "utility."

Chapters VII and IX deal with "strictly competitive games"—i.e., conflicts between two individuals whose aims are in direct opposition, so that there would be no point in cooperating. Conceptually this is the relatively easy part of game theory; under the assumption that both players are rational, it is possible to give good, convincing advice in such situations.[5] Chapter VIII describes the important concept of "strategy"; this term is used in game theory in a sense somewhat different from (though related to) its everyday sense.

Chapter X deals with non-cooperative games that are not strictly competitive. Here "non-cooperative" does not mean that the players do not want to cooperate or that it would not be advantageous for them to do so, but that they are prevented from cooperating because there is no possibility of communicating or no enforcement mechanism for

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[4] The word "politician," which would have been more fitting, has become a "no-word."

[5] I feel obliged to express strong disagreement with the contents of the last two sections of Chapter IX (pp. 161-65), and especially with the first of these (pp. 161-63), in which the author philosophizes about the advice that is given by game theory to the players in a strictly competitive game. In fact, the reasoning behind this advice is a good deal sounder than is apparent from the discussion in these sections, which is based on serious misunderstandings of utility theory.

agreements.[6] Chapters XI and XII describe cooperative games—i.e., games in which cooperation is both profitable and feasible; here the main problem is how to divide the spoils, and this can be a sore problem indeed. Chapter XI deals with arbitration procedures—i.e., "fair" methods of dividing the spoils ("fair" in relation to the relative strengths of the players); Chapter XII is concerned with the problems of coalition formation.

Chapter XIII deals with experimentation in game theory, of which more below; Chapter XIV discusses critiques of game theory, by other authors and by Rapoport himself. This ends the "games" part of the book.

The previous four paragraphs were meant to serve as a table of contents only. Obviously this is no place to review the theory of games; for this the reader is referred to the book itself, which in any event does the job a good deal better than this reviewer could do it. Instead, I will discuss three questions of a general nature that are raised by a reading of the book: the relevance of experimental work to a normative theory, the role of mathematics in a mathematical model, and the applicability of game theory (or mathematical models in general) to political science (or extremely "inexact" sciences in general).

III

A considerable amount of "experimental work in game theory" has been performed, and some of it is described in the book before us. According to the above characterization of game theory as normative, the phrase in quotes is a contradiction in terms, and it is no wonder that many such studies yield results "contrary to the conclusions of game theory." Suppose you were interested in enclosing a maximum area with a given length of fencing. You would not poll a hundred farmers to find out how to do it; you would work it out by the methods of geometry. If you did poll the farmers, and they all gave the wrong answer, what relevance would this have to your problem? It would say something of interest about the farmers, but nothing about your problem.

Rapoport reports on an experiment (performed by Vinacke and Arkoff) in which three players were given a sum of money to divide as they saw fit; the mode of division was decided by majority vote, but each of the players had a different number of votes available to him (in much the same way that each state political boss has a different

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[6] See the end of section III below for an example of this (the overproducing farmers).

number of votes available to him at a political convention).[7] Six combinations were tried, as follows:

Players	A B C	A B C	A B C	A B C	A B C	A B C
Number of votes of each player	1 1 1	3 2 2	1 2 2	3 1 1	4 3 2	4 2 1
Combination No.	I	II	III	IV	V	VI

Now, in spite of the window dressing, there are clearly only two different kinds of games here. In I, II, III, and V, any two players form a majority; all these games are completely symmetrical, and all players are equally "strong" in these games. In IV and VI player A forms an absolute majority alone, and both B and C are completely powerless. One does not have to be a game theorist to realize this; it is just common sense.

The results were quite fantastic. Suffice it to say that in a considerable number of games of types IV and VI, player A did *not* simply take all the proceeds for himself, but formed a coalition with one of the other players. In games of types I, II, III, and V, the players were also guided by the "window dressing" rather than by their actual strength. Incidentally, Vinacke and Arkoff were well aware that their subjects were behaving irrationally. Their study was a psychological-sociological one—i.e., it was descriptive and had no normative overtones whatsoever.

When then is the relevance—if any—of experimentation to game theory? Rapoport suggests that a descriptive theory be attempted, parallel to social physics but dealing with cooperation and conflict in small groups[8] (such as in the above experiment). He asks whether general probabilistic laws could be observed governing behavior in game-like situations, and whether such observed laws could be derived from some set of basic postulates—as Kepler's observed laws for the motion of the planets can be derived from Newton's basic laws of motion. According to Rapoport, "the experimental scientist should take over to explore the norms actually operating in human affairs" (p. 225).[9] There is much to be said for this point of view, but the

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[7] The game was disguised somewhat by means of a set of rather complicated rules which, however, were quite transparent. Cf. W. E. Vinacke and A. Arkoff, "An Experimental Study of Coalitions in the Triad," *American Sociological Review*, xxii (1957), pp. 406-15.

[8] These need not be groups of people, but can also, for example, be groups of nations.

[9] If one interprets "norm" as in "normative"—i.e., as a standard according to which people *should* behave—then the phrase "norms actually operating" is devoid of sense. A physician was once asked at a murder trial how long a normal person lives. His answer was that a normal person doesn't die; medically, there is something wrong with everyone who dies.

program must be executed with great caution; there are many pit-falls. First, it must be recognized that such an experimental, descriptive science would in no sense constitute a reorientation or a replacement of classical (i.e., normative) game theory, and indeed the two would not be related. In view of this, great pains should be taken not to introduce any rationality postulates into such a theory, but only to fit the postulates to the observations. Next, it stands to reason that the behavior of students in small groups, playing games for insignificant amounts of money with little time or incentive to think things through, is in no sense representative of, say, the behavior of nations in the international arena. It is absurd to suppose that when important decisions are to be made, the players of the international game would commit the comic errors that seem to be characteristic of the subjects in game experiments.[10] If one wants to "explore the norms actually operating in human affairs," then one must decide whether to study trivial or significant affairs, and take care not to reach conclusions of significance from observations of triviality.

For example, it turns out that the theory of games, normative though it is, is at least in parts not so far off the beam as a *description* of international behavior. One of the important notions of game theory is that of "equilibrium"—a situation in which no individual player can improve his position by unilateral action alone. There are games[11] in which rational behavior by the players leads inexorably to equilibrium outcomes that are close to the *worst possible* for *all* players; an enlightening practical example of this kind is offered by Luce and Raiffa: Consider a market with many small producers (such as farmers), where the total productive capacity far exceeds the demand. It is to the interest of all the producers to limit production. Nevertheless, the actions of any one producer cannot affect the price, so no matter what the other producers decide, it is to the advantage of each individual producer to produce as much as he can. It follows that the only equilibrium is the one at which each producer produces at his maximum capacity, and therefore all go into bankruptcy. Rapoport dismisses these equilibria because they are "not very relevant to the interests of the players" (p. 224). In fact, neither is the cold war very relevant to the interests of the United States and the Soviet Union; it apparently represents a situation close to equilibrium (in the above technical sense), which would explain its stubborn persistence.[12] This

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[10] Though sometimes one is forced to wonder.

[11] Such as the Prisoner's Dilemma.

[12] Professor R. D. Luce suggested at the Princeton University Conference on Arms

suggests that normative considerations of individual rationality are not entirely irrelevant to "human affairs" as actually conducted, and cannot be lightly dismissed—whether the conclusions be pleasant or unpleasant.

IV

I have quoted Rapoport's suggestion that experimental evidence be used to set up a descriptive theory parallel to the normative theory of games. It is my belief that experimental work can be of direct relevance to normative game theory itself, but before I can say how, we must discuss our second point—namely, the role of mathematics in a mathematical model. I believe it was Bertrand Russell who characterized mathematics as the subject in which "we don't know what we're talking about,[13] nor whether what we're saying is true." The first phrase refers to the fact that the basic entities of mathematical theories are undefined terms, like "point" in geometry. What he meant by the second phrase is that mathematics deals only with *deductions*, so that one can never know whether the conclusion is true unless one knows about the hypothesis. In order to make applied mathematics out of mathematics, one must give real-life interpretations to the basic entities of the theory, and one must make real-life assumptions. By means of mathematics alone, one can never "prove" anything, nor even recommend anything, concerning a subject that has to do with the real world. The assumptions of game theory are "rationality" assumptions—an example is "If you prefer[14] possibility a to b, and possibility b to c, then you prefer a to c"; here the conclusion must be interpreted to mean that you *should* prefer a to c if you want to be "consistent," it being a known fact that people do not always behave in accordance with this assumption. The distinction between normative and descriptive theories lies in whether the assumptions are taken as recommendations or as truths, but does not lie in the mathematics itself. Now the simple-minded kind of assumption of which the above is an example suffices to build a theory that enables us to give recommendations for action in "strictly competitive games" (cf. section II above); but most kinds of conflict-cooperation situations are too complex and subtle to be handled satisfactorily in this way, and other, more contro-

Control and Limitation in October 1960 that it might be worthwhile to look for more benign equilibria in this situation.

[13] No applause, please.

[14] The term "prefer" is an example of an undefined entity in the mathematical model; to give the theory real-life significance, it must be translated (into its everyday meaning).

versial assumptions must be introduced in order to reach results. It is not really the mathematician's business to resolve the controversy by choosing a set of assumptions, but it usually has been left for him to do so.[15] In no case, however, can one speak (as Rapoport does) of a "purely mathematical solution" or of "relationships dictated by mathematical considerations"; at most one can say that one does not consider a certain set of assumptions useful, though they may lead to a mathematically elegant theory. I should emphasize here that beyond the field of strictly competitive games, game theory consists not of one monolithic body of work, but of various "theories" based on different sets of assumptions.

Traditionally, basic assumptions in game theory have been based on *individual* rationality; that is to say, though the individual may have had to modify his demands by the realities of his situation, still the basic assumptions have always involved each player's maximizing his payoff—i.e., playing to get as much as he can. One of the most provocative of Professor Rapoport's suggestions is that we examine consequences of assumptions which set the welfare of society above the welfare of the individual—more specifically, which set the moderate welfare of all individuals above the extreme welfare of oneself. Referring to the overproducing farmers whom we discussed above, this approach would lead to voluntary acreage restrictions on the part of all the farmers, *without enforcement or even previous communication.*[16] This may sound unduly utopian, but the author cites examples of similar assumptions in everyday (American) life: "The argument that a single vote 'makes no appreciable difference' is countered with 'Yes, but if everyone thought so, the will of the collective would find no expression.' This is the rationale behind any cooperative effort. It is conceivable that the (minimal) danger inherent in submitting to vaccination actually exceeds the (still more minimal) danger of remaining unvaccinated. But if *everyone* minimized the danger to himself alone, everyone would be subjected to the much greater danger of a smallpox epidemic. . . . The reason this appears paradoxical, I suspect, is that we have been too long accustomed to the uncritical acceptance of the laissez faire principle, namely, that a totality of individuals seek-

[15] The significant exception is the collaboration of the economist Oskar Morgenstern with the mathematician John von Neumann in their book, *Theory of Games and Economic Behavior*, Princeton, N.J., 1944.

[16] Government-imposed or otherwise enforced acreage restrictions would be nothing new in game theory; they are the result of "arbitration" or "cooperative" procedures. The novelty of Rapoport's suggestion lies in that he obtains this result in the *non-cooperative* case.

ing their respective self-interests by *shortsighted*[17] calculations actually will move toward the realization of this self-interest by the operation of economic laws derived from the assumptions of a free competitive market economy" (p. 177). This is a powerfully appealing idea, though it is not clear to me how it can be turned into a general theory. Unfortunately, it has no application to international politics; there self-interest (enlightened or otherwise) reigns supreme.

Let us now return to the subject of experimentation. In spite of what was said above about the inherent inapplicability of experimental evidence to a normative theory, there is a feeling among those interested in games that the theory is too much up in the air, too detached from reality. Game-theorists sit in office chairs and decide that such and such a procedure is "rational," much like the popular image of Aristotle sitting in an armchair and deciding that heavy bodies fall faster than light ones. In science, experimentation is generally used to test theories. My feeling is that in game theory, under the right conditions, experimentation can be used for the converse purpose—that is, to suggest reasonable starting points (i.e., assumptions). The "right conditions" are that the subjects be intelligent, that the decisions they are asked to make be important to them, and that they be given time to think them over—plenty of it, as behooves important decisions. This would serve three related purposes: it would enable investigators to get a "feel" for real-life conflicts; it might bring to light important factors that the theorists have overlooked; or possibly it could indicate entirely new approaches. Alternatively, experimentation *could* be used to *test* a theory of conflict, in the following way: all the subjects would be made thoroughly aware of the assumptions and conclusions being tested, and also aware that the other subjects are aware; in addition, the "right conditions" referred to above should obtain. If the theory is then followed, we may feel justified in calling it "empirically consistent" (although nothing has been "proved" in the traditional sense of empirical science); if not, then this indicates a serious flaw in the proposed theory, and a follow-up investigation should be conducted to see what "went wrong." This is of course radically different from experimental procedures for the confirmation of descriptive theories, in which informing the subjects of the proposed theory would be unforgivable. For normative theories, on the other hand, the above method seems eminently suitable.

Returning to the first application of experimentation to game theory,

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[17] This word doesn't really belong there. The point is not shortsightedness, but selfishness; also in the long run it is better for every *individual* not to be vaccinated.

as previously—i.e., such advice has not actually been given to any large extent (outside of the military field). Political situations, on the other hand, seem too amorphous for mathematical treatment; personalities take on a decisive importance,[19] the significant factors cannot be assessed and are too numerous, the unanticipated (as distinguished from the merely unexpected) is always happening, the relationships are too involved and difficult to define. Often the problem itself is quite vague, with the alternatives unclearly stated or not stated at all; on the contrary, the problem may be to *find* the alternatives. It is useless to think of game theory in connection with questions like "How can we gain the confidence of the new African nations?" Such questions simply lack the framework necessary for mathematical reasoning.

But we must not lightly dismiss the mathematical approach, even though at first it seems inapplicable. Mathematics has a way of becoming useful in the most unlikely places. For example, in complex decisions that must be made intuitively because of the lack of appropriate systematic tools, a high degree of haphazardness is often inescapable. In such situations mathematical models can sometimes be developed by suppressing all but a few of the important factors, even though the suppressed factors may be highly significant. The suppression of admittedly significant factors seems unallowable; but the choice may be between the mathematical model, which is bad because it suppresses *some* factors, and the intuitive decision, which is terrible because it is utterly haphazard, suppressing almost all the significant factors.[20]

An example—which has nothing to do either with game theory or with politics—is clinical experimentation in medicine, to determine the desirability of a given treatment. At first glance this seems utterly inaccessible to the methods of mathematical statistics: random sampling is difficult, if not impossible; there is no uniformity among the subjects as regards effectiveness of the treatment or seriousness of the side-effects; the importance of side-effects differs from patient to patient (should a seriously abscessed arm be amputated? It makes a difference whether the patient is a pianist or a professor); when a promising new treatment is being tested, ethical considerations (which

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[19] We are discussing decision-making in a specific situation; I think even Tolstoyites (and Marxists) would agree about the importance of personality factors in the short run.

[20] This recalls the man who, as he was emerging from the subway, complained to his companion: "Damn it, it's raining again"; whereupon the companion answered, "Better than nothing."

(i.e., as a method for suggesting appropriate assumptions), it would appear that political history and business records should serve as a good "laboratory"—at least, if used selectively. Indeed, I feel that a study of these sources from a game-theoretic viewpoint should be undertaken. The trouble with these sources is that they are often too "messy" for a neat mathematical framework; and this brings us to the last point we wanted to discuss, the applicability of game theory to politics.

V

I must start by confessing my ignorance of political theory. The following should therefore be taken as the tentative remarks of a layman, based on a cursory reading of the newspapers and some small experience with the application of mathematical methods to areas (other than politics) in which subjective factors loom large. Also, it constitutes a highly personal view, not necessarily shared by others interested in the theory of games.

Game theory can be used in two ways: first, to get a general, qualitative "feel" for a situation, like that conveyed by the equilibrium nature of the cold war, as mentioned above;[18] second, to give specific advice in a specific situation. The importance of the first kind of application should not be underestimated; much of theory in the social sciences is of this nature. In the case of political science or, more accurately, of politics, game theory is in principle certainly applicable in this first sense. I say "in principle" because, aside from remarks here and there (some of which are quite valuable), I know of no serious, systematic attempt to apply game-theoretic reasoning to political science. Potentially, game-theoretic insights could assume a significance in the conduct of international affairs similar to that of a knowledge and understanding of history.

The second kind of application—specific advice in a specific situation—is another matter. Such an application of mathematical methods, be it in physics or politics, demands that the significant factors be apparent and in some sense measurable. Above all, it demands that the specific situation in question be "structured," that it have some handle that the mathematician can grasp. Now these demands are more or less satisfied in certain military, business, and economic problems, and indeed game theory can in principle give specific advice in many such real-life problems; here "in principle" means the same

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[18] Game theory abounds in such insights. For those familiar with the von Neumann-Morgenstern solutions, another example is that of discriminatory solutions; still another one is Shapley's index of power.

are difficult, if not impossible, to assess mathematically in a meaningful way) play havoc with the rigid controls demanded by statistical theory; costs (in money and time) are certainly not irrelevant, but to what extent should they be taken into account? Statistical methods can be applied only if fairly arbitrary decisions are made on the above questions, or if they are ignored completely. Nevertheless statistics has had considerable success in this field; indeed the only alternative is chaos, each physician deciding for himself what treatment to administer, on what can only be a haphazard, intuitive basis.

I have presented this dichotomy as though the application of statistical methods were obviously preferable. But there *is* something to be said for the other side. "Chaos" means that all clinical researchers publish case histories only, with no attempt at planned experiments. A physician who read all this literature and digested and remembered it all might conceivably do better by treating a particular, individual patient on the basis of an intuitive judgment than by following recommendations based on a statistically planned experiment.

To sum up: Game theory should definitely be applicable to political *theory*. As for game-theoretic advice in specific political situations, at present I see no prospect for anything really useful in this direction. But others disagree, and this kind of application could become a reality in the future. Possibly a start should be made on something like labor-management bargaining, which is similar in some ways to international bargaining, yet is more sharply defined and standardized, and hence more amenable to mathematical treatment (note that I have not said "amenable," only "more amenable"). In any case, though it may be difficult, an effort to develop tools for the application of game theory to specific political problems would be highly interesting and worthwhile.