Exercises and problems II:

1. (a)(2-) Describe the face ring of the square and the Octahedron

   (b) (2?) Carry out the definition of the structure sheaf and the IH sheaf for the standard square in the plane $conv\{(1,1),(-1,1),(1,-1),(-1,-1)\}$.

   (c) (2??) The same for the standard 3-dimensional cube.

2. (a) (2+) Show that for a 4-dimensional polytope P $g_2(P) = g_2(P^*)$.

   (b) (3-) Extends for higher dimensional polytopes

   (c) (3) Extend further for higher dimensional polytopes.

   (d) (3) Find a different extension for higher dimensional polytopes.

   (e+f) (3+ and 4) Find an algebraic and an geometric reason in terms of intersection homology

3. (a) (2) Prove that if P is a polytope and F is a face of P then $g_1(F) \leq g_1(P)$.

   (b) (2) Let P be a simple polytope and F a face of P. show that $\bar{h}_k(F) \leq \bar{h}_k(P)$. ($\bar{h}_k(P)$ is the dual-h vector as defined in terms of linear objective functions.)

   (c) (2+) Show that for a Cohen-Macaulay complex K and a link K' of a vertex in K, $h_j(K') \leq h_j(K)$

4. (4-) It is known that the cohomology ring of a toric variety associated to a simplicial polytope is isomorphic (the grading is doubled) to the face ring of the boundary complex of P. Therefore the cohomology is generated as a ring by degree 2 elements. Find a geometric reason.

5. (a) (4) (The major open problem for the first half of the seminar.) Let L be a graded atomic Euleruan lattice which represent a regular CW-decomposition of a $(d-1)$-dimensional sphere. Show that $(g_0(L), g_1(L), \ldots, g_{[d/2]}(P))$ is an M-sqquence.

   (b) (4-) Shoe that for a polyhedral 3-sphere K, $g_2(K) \geq 0$.